14.9.3

$$A = \begin{vmatrix} 2 & 0 & 0 \\ 1 & 7 & 3 \\ 5 & 4 & 3 \end{vmatrix}$$

$$Ax = \lambda x$$

$$\begin{vmatrix} 2 & 0 & 0 \\ 1 & 7 & 3 \\ 5 & 4 & 3 \end{vmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix}$$

$$2x_1 = \lambda x_1 \longrightarrow 2x_1 - \lambda x_1 = 0$$
$$x_1 + 7x_2 + 3x_3 = \lambda x_2 \longrightarrow x_1 + 7x_2 - \lambda x_2 + 3x_3 = 0$$
$$5x_1 + 4x_2 + 3x_3 = \lambda x_3 \longrightarrow 5x_1 + 4x_2 + 3x_3 = 0$$

$$(2-\lambda)x_1 + 0 + 0 = 0$$
$$x_1 + (7-\lambda)x_2 + 3x_3 = 0$$
$$5x_1 + 4x_2 + (3-\lambda)x_3 = 0$$

$$\begin{vmatrix} (2-\lambda) & 0 & 0 \\ 1 & (7-\lambda) & 3 \\ 5 & 4 & (3-\lambda) \end{vmatrix} = 0$$

$$(2-\lambda)(7-\lambda)(3-\lambda) - 12(2-\lambda) = 0$$
$$(2-\lambda)[(7-\lambda)(3-\lambda) - 12] = 0$$
$$2-\lambda = 0$$
$$\lambda = 2 \quad \text{ou} \quad (7-\lambda)(3-\lambda) - 12 = 0$$
$$21 - 7\lambda - 3\lambda + \lambda^2 - 12 = 0$$
$$\lambda^2 - 10\lambda + 9 = 0$$

$$\Delta = b^2 - 4ac$$
$$\Delta = (-10)^2 - 4 \cdot 1 \cdot 9$$
$$\Delta = 100 - 36$$
$$\Delta = 64$$

$$\lambda = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$\lambda = \frac{10 \pm 8}{2} \quad \lambda_1 = 9, \quad \lambda_2 = 1$$

$\lambda = 2$

$$\begin{cases} 0x_1 + 0x_2 + 0x_3 = 0 \; * \\ x_1 + 5x_2 + 3x_3 = 0 \longrightarrow x_1 = -5x_2 - 3x_3 \\ 5x_1 + 4x_2 + x_3 = 0 \longrightarrow x = \dfrac{-4x_2 - x_3}{5} \end{cases}$$

$$-5x_2 - 3x_3 = \frac{-4x_2 - x_3}{5} \rightarrow -25x_2 - 15x_3 + 4x_2 + x_3 = 0$$

$$-21x_2 - 14x_3 = 0 \rightarrow x_2 = \frac{-14x_3}{21} \rightarrow x_2 = \frac{-2x_3}{3}$$

$$* \; x_1 = -5\left(-\frac{2}{3}x_3\right) - 3x_3$$
$$= \frac{10}{3}x_3 - 3x_3$$
$$= \frac{10x_3 - 9x_3}{3} \rightarrow x_1 = \frac{x_3}{3}$$

$$\rightarrow (x_1, x_2, x_3) = \left(\frac{x_3}{3}, \frac{-2x_3}{3}, x_3\right)$$

Supondo $x_3 = 1$

$$V_1 = \left(\frac{1}{3}, -\frac{2}{3}, 1\right) \text{ e } \lambda = 2$$

11.83

a) $3x - y = 7$
$-6x + 2y = -14$ $\quad \begin{vmatrix} 3 & -1 \\ -6 & +2 \end{vmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ -14 \end{pmatrix}$

b) $3x - y = 5$
$-6x + 2y = 4$ $\quad \begin{vmatrix} 3 & -1 \\ -6 & +2 \end{vmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{vmatrix} 5 \\ 4 \end{vmatrix}$

c) $3x - y = 0$
$-6x + 2y = 0$ $\quad \begin{pmatrix} 3 & -1 \\ -6 & +1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

d) $3x - y = 5$
$-6x + y = 4$ $\quad \begin{pmatrix} 3 & -1 \\ -6 & +1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$

e) $3x - y = 0$ $\quad a \neq 0$
$-6x + y = 0$ $\quad b \neq 0$ $\quad \begin{pmatrix} a & b \\ a & -b \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

f) $ax + by = 0$ $\quad a \neq 0$
$ax - by = 0$ $\quad b \neq 0$

a) $\begin{pmatrix} 3 & -1 \\ -6 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ -14 \end{pmatrix}$

$\begin{pmatrix} 3x & -y \\ -6x & -2y \end{pmatrix} = \begin{pmatrix} 7 \\ -14 \end{pmatrix}$

$3x - y = 7$
$3x = 7 + y$
$x = \frac{7+y}{3}$

$-6x - 2y = -14$
$-6x = -14 + 2y$
$-x = -\left(\frac{-14+2y}{6}\right)$

$\frac{7+y}{3} = -\frac{14+2y}{6}$

infinitas soluções!

$\boxed{0 = 0}$

b) não tem solução

c) $3x - y = 0$
$-6x + 2y = 0$ $\quad \begin{pmatrix} 3 & -1 \\ -6 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$,

$6x + 2y = (-2)(3x - y)$
→ dependência linear
$y = 3x$ infinitas soluções

d) $\begin{cases} 3x - y = 5 \\ -6x + y = 4 \end{cases}$
$-3x = 9$
→ $-3x = 9$
$x = -3$

$y = 3x - 5$
$y = 3(-3)(-5)$
$y = -14$

Uma única solução

e) $\begin{cases} 3x - y = 0 \\ -6x + y = 0 \end{cases}$
$-3x = 0$
$y = 0$
→ Uma única solução!

f) $\begin{cases} ax + by = 0 \\ ax - by = 0 \end{cases}$
$2ax = 0$

$x = 0$
$y = 0$

Uma única solução

$$A^{-1} = \frac{1}{6}\begin{pmatrix} -2 & 4 & 1 \\ -6 & 6 & 3 \\ 12 & -12 & -3 \end{pmatrix} = \begin{bmatrix} -\frac{2}{6} & \frac{4}{6} & \frac{1}{6} \\ \frac{-6}{6} & \frac{6}{6} & \frac{3}{6} \\ \frac{12}{6} & -\frac{12}{6} & -\frac{3}{6} \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} & \frac{1}{6} \\ -1 & 1 & \frac{1}{2} \\ 2 & -2 & -\frac{1}{2} \end{bmatrix}$$

Se $x = A^{-1} \cdot y$

$$x = \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} & \frac{1}{6} \\ -1 & 1 & \frac{1}{2} \\ 2 & -2 & -\frac{1}{2} \end{bmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \qquad x = \begin{bmatrix} -\frac{y_1}{3} + \frac{2y_2}{3} + \frac{y_3}{6} \\ -y_1 + y_2 + \frac{y_3}{2} \\ 2y_1 - y_2 - \frac{y_3}{2} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\therefore \quad x_1 = \frac{-y_1}{3} + \frac{2y_2}{3} + \frac{y_3}{6}$$

$$x_2 = -y_1 + y_2 + \frac{y_3}{2}$$

$$x_3 = 2y_1 - y_2 - \frac{y_3}{2}$$

14.8.1

$a = (3, -1, 5, 2)$
$b = (0, 0, 5, 2)$
$c = (3, -1, 0, 0)$

$a\lambda_1 + b\lambda_2 + c\lambda_3 = 0$

$$\lambda_1(3,-1,5,2) + \lambda_2(0,0,5,2) + \lambda_3(3,-1,0,0) = (0,0,0,0)$$

$$(3\lambda_1, -\lambda_1, 5\lambda_1, 2\lambda_1) + (0, 0, 5\lambda_2, 2\lambda_2) + (3\lambda_3, -\lambda_3, 0, 0) = (0,0,0,0)$$

$$(3\lambda_1 + 3\lambda_3, -\lambda_1 - \lambda_3, 5\lambda_1 + 5\lambda_3, 2\lambda_1 + 2\lambda_2) = (0,0,0,0)$$

1. $3\lambda_1 + 3\lambda_3 = 0$
2. $-\lambda_1 - \lambda_3 = 0$
3. $5\lambda_1 + 5\lambda_2 = 0$
4. $2\lambda_1 + 2\lambda_2 = 0$

1) $\lambda_1 = -\lambda_3$ 3) $\lambda_1 = -\lambda_2$

2) $\lambda_1 = -\lambda_3$ 4) $\lambda_1 = -\lambda_2$

$\therefore \lambda_1 = -\lambda_3 = -\lambda_2$

Nessas condições, são dependentes

14.7.9 $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ $A = \begin{bmatrix} 3 & 0 & 1 \\ 3 & -1 & 0 \\ 0 & 4 & 2 \end{bmatrix}$ $Y = \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix}$ $Ax = Y$ $\underbrace{3\times 3 \times 3\times 1}_{3\times 1}$

$$3x_1 + x_3 = Y_1 = 1$$
$$3x_2 - x_2 = Y_2 = 1 \rightarrow 2x_2 = 1 \rightarrow x_2 = \frac{1}{2}$$
$$4x_2 + 2x_3 = Y_3 = 1$$

$$4 \cdot \frac{1}{2} + 2x_3 = 1 \rightarrow 2x_3 = -1 \qquad x_3 = -\frac{1}{2}$$

$$3x_1 - \frac{1}{2} = 1 \rightarrow 3x_1 = \frac{3}{2} \rightarrow x_1 = \frac{1}{2}$$

$\therefore$ $x = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$ e $Y = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

[redacted]

$Ax = Y$ $\qquad$ $Ax = Y$

$x = A^{-1}Y$ $\qquad$ $x = \frac{Y}{A} \rightarrow A \cdot Y \rightarrow \frac{1}{x^{-1}} = x^{-1}$

$\begin{vmatrix} -1 & 0 \\ 4 & 2 \end{vmatrix} = -2 - 0 = -2$ $\qquad$ $\begin{vmatrix} 3 & 0 \\ 0 & 2 \end{vmatrix} = 6 - 0 = 6$

$\begin{vmatrix} 3 & -1 \\ 0 & 4 \end{vmatrix} = 12 - 0 = 12$ $\qquad$ $\begin{vmatrix} 0 & 1 \\ 4 & 2 \end{vmatrix} = 0 - 4 = -4$

$\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix} = 6 - 0 = 6$ $\qquad$ $\begin{vmatrix} 3 & 0 \\ 0 & 4 \end{vmatrix} = 12 - 0 = 12$

$\begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} = 0 - (-1) = 1$ $\qquad$ $\begin{vmatrix} 3 & 1 \\ 3 & 0 \end{vmatrix} = 0 - 3 = -3$

$\begin{vmatrix} 3 & 0 \\ 3 & -1 \end{vmatrix} = -3 - 0 = -3$

$$\begin{pmatrix} -2 & 6 & 12 \\ -4 & 6 & 12 \\ 1 & -3 & -3 \end{pmatrix} \rightarrow \begin{pmatrix} -2 & -6 & 12 \\ 4 & 6 & -12 \\ 1 & 3 & -3 \end{pmatrix} \rightarrow \begin{pmatrix} -2 & 4 & 1 \\ -6 & 6 & 3 \\ 12 & -12 & -3 \end{pmatrix}$$

$\det A = \det \begin{vmatrix} 3 & 0 & 1 \\ 3 & -1 & 0 \\ 0 & 4 & 2 \end{vmatrix} \rightarrow (-6 + 0 + 12) - (0 + 0 + 0) = +6$

14.6.14

$$A = \begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix} \quad B = \begin{pmatrix} -1 & 3 \\ 1 & 5 \end{pmatrix} \quad \det(AB) = \det A \cdot \det B$$

$2\times2 \times 2\times2$
2×2

$\det A = 3 - 8 = -5 \quad \det B = -5 - 3 = -8 \quad \therefore \det(AB) = \det A \cdot \det B = -5 \cdot -8 = $

$$AB \begin{pmatrix} -3+2 & 9+10 \\ -4+1 & 12+5 \end{pmatrix} = \begin{pmatrix} -1 & 19 \\ -3 & 17 \end{pmatrix}$$

$\det(AB) = -17 + 57 = 40$

14.7.3

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 4 & 2 \\ 3 & 0 & 1 \end{bmatrix} \begin{matrix} 3 & -1 \\ 0 & 4 \\ 3 & 0 \end{matrix}$$

$-(+0+0+0)+12-6 = 6 \rightarrow \therefore A$ é inversível

$$\begin{bmatrix} 3 & -1 & 0 \\ 0 & 4 & 2 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$3a - d = 1 \qquad 3b - e = 0 \qquad 3c - f = 0$
$4d + 2g = 0 \qquad 4e + 2h = 1 \qquad 4f + 2i = 0$
$3a + g = 0 \qquad 3b + h = 0 \qquad 3c + i = 1$

$3a - 1 = d$
$4(3a-1) + 2g = 0$
$g = -3a$
$4(3a-1) - 6a = 0$
$12a - 4 - 6a = 0$
$6a = 4$
$a = \frac{2}{3}$, $d = 1$
$g = -2$

$3b = e \quad h = -3b$
$4 \cdot 3b + 2(-3b) = 1$
$12b - 6b = 1$
$6b = 1$
$b = \frac{1}{6}$,
$e = \frac{1}{2}$ e $h = -\frac{1}{2}$

$3c = f \quad 3c - 1 = -i$
$4 \cdot 3c - 2(3c-1) = 0$
$12c - 6c + 2 = 0$
$6c = -2$
$c = -\frac{1}{3}$,
$f = -1$ e $-1 = 1 - i$
$i = 2$

$$\therefore A^{-1} = \begin{bmatrix} \frac{2}{3} & \frac{1}{6} & -\frac{1}{3} \\ 1 & \frac{1}{2} & -1 \\ -2 & -\frac{1}{2} & 2 \end{bmatrix}$$

14.6.1

a) $\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix}$

$3.5 - 2.4 = 7$

b) $\begin{vmatrix} 3 & 1 \\ -2 & 4 \end{vmatrix}$

$3.4 + 2.1 = 10$

c) $\begin{vmatrix} 18 & 11 \\ 6 & -2 \end{vmatrix}$

$-36 - 66 = -102$

14.6.7

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 8 \\ 3x_1 + x_2 - x_3 &= 1 \\ 5x_1 - 3x_2 + 2x_3 &= -3 \end{aligned}$$

Pelo cálculo do jstc:

$x_1 = -10/7$
$x_2 = -103/7$
$x_3 = -20$

$$A = \left|\begin{array}{ccc|cc} 1 & -2 & 1 & 1 & -2 \\ 3 & 1 & -1 & 3 & 1 \\ 5 & -3 & 2 & 5 & -3 \end{array}\right.$$

$-(5+3-12)+2+10-9$

$\det A = 3+4 = 7$

$$Ax = \left|\begin{array}{ccc|cc} 8 & -2 & 1 & 8 & -2 \\ 1 & 1 & -1 & 1 & 1 \\ -3 & -3 & 2 & -3 & -3 \end{array}\right.$$

$-(-3+24-4)+16-6-3$

$\det(Ax) = 7 - 17 = -10$

$$Ay = \left|\begin{array}{ccc|cc} 1 & 8 & 1 & 1 & 8 \\ 3 & 1 & -1 & 3 & 1 \\ 5 & -3 & 2 & 5 & -3 \end{array}\right.$$

$-(5+3+48)+2-40-9$

$\det(Ay) = -47 - 56 = -103$

$$Az = \left|\begin{array}{ccc|cc} 1 & -2 & 8 & 1 & -2 \\ 3 & 1 & 1 & 3 & 1 \\ 5 & -3 & -3 & 5 & -3 \end{array}\right.$$

$-(40-3+18)-3-10-72$

$\det(Az) = -85 - 55 = -140$

$$x = \frac{\det(Ax)}{\det A} = -\frac{10}{7}$$

$$y = \frac{\det(Ay)}{\det A} = -\frac{103}{7}$$

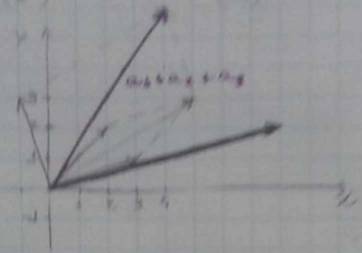
$$z = \frac{\det(Az)}{\det A} = -\frac{140}{7} = 20$$

14.4.1

$a_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ $a_2 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ $a_3 = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

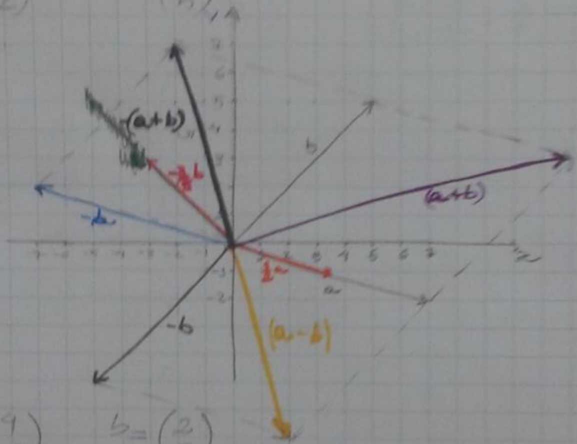
a) $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$

b)



14.4.5

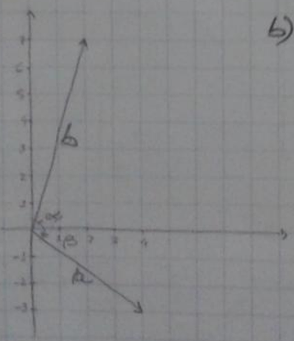
$a = \begin{pmatrix} 7 \\ -2 \end{pmatrix}$ $b = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$



14.4.7

$a = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ $b = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$

a)



b)

$|a| = \sqrt{7^2 + 2^2} = \sqrt{53}$

$|b| = \sqrt{4^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

$(\alpha + \beta) = ?$ $tg\,\beta = \frac{3}{4}$ $tg\,\alpha = \frac{7}{2}$

$(\alpha + \beta) = arctg\left(\frac{3}{4}\right) + arctg\left(\frac{7}{2}\right)$

$= 36{,}87^\circ + 74{,}05^\circ = 110{,}92^\circ$

14.4.13

$u^{\to} \cdot v^{\to} = (-6, -3, 0) \to$ Soma $= -9$ $v^{\to} \cdot w^{\to} = (-10, 6, 4) \to$ Soma $= 0$

$u^{\to} \cdot w^{\to} = (15, -2, 0) \to$ Soma $= 13$ $\therefore$ $v^{\to}$ e $w^{\to}$ são vetores ortogonais

14.2.24

$$A = \begin{pmatrix} 4 & -2 \\ -5 & 3 \end{pmatrix} \qquad B = \frac{1}{2}\begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix} \qquad AB = I \text{ e } BA = I$$

$$\begin{pmatrix} 4 & -2 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} 3/2 & 1 \\ 5/2 & 2 \end{pmatrix} = \begin{pmatrix} 6-5 & 4-4 \\ -15/2+15/2 & -5+6 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 3/2 & 1 \\ 5/2 & 2 \end{pmatrix}\begin{pmatrix} 4 & -2 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} 6-5 & -3+3 \\ 10-10 & -5+6 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\therefore\ AB = I$ e $BA = I$, $A = B^{-1}$ e $B = A^{-1}$

14.2.25 $A = \frac{1}{5}\begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix}$ $\qquad A^tA = I$ e $AA^t = I$

$A \cdot A^t = A^tA = I \rightarrow$ matriz ortogonal

$$\begin{pmatrix} 3/5 & 4/5 \\ -4/5 & 3/5 \end{pmatrix} = A \qquad A^t \cdot A = \begin{pmatrix} (3/5)^2 + (4/5)^2 & 3/5 \cdot 4/5 - 4/5 \cdot 3/5 \\ 4/5 \cdot 3/5 - 4/5 \cdot 3/5 & (4/5)^2 + (3/5)^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{pmatrix} = A^t \qquad A \cdot A^t = \begin{pmatrix} \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 & -\frac{12}{5} + \frac{12}{5} \\ -\frac{12}{25} + \frac{12}{25} & +\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\therefore\ A^tA = AA^t = I$

14.2.26

$$A = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{\sqrt{2}}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} \qquad A^t = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{\sqrt{2}}{2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$$

$$A^tA = \begin{bmatrix} \frac{2}{4} + \frac{2}{4} + 0 & -\frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4} + 0 & -\frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4} + 0 \\ -\frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4} + 0 & +\frac{1}{4} + \frac{1}{4} + \frac{2}{4} & +\frac{1}{4} + \frac{1}{4} - \frac{2}{4} \\ -\frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4} + 0 & +\frac{1}{4} + \frac{1}{4} - \frac{2}{4} & +\frac{1}{4} + \frac{1}{4} + \frac{2}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore\ A^tA = I$

14.2.18 $A = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$ $B = \begin{pmatrix} -1 & 2 \\ 3 & 2 \end{pmatrix}$ $C = \begin{pmatrix} 5 & 1 \\ 1 & 7 \end{pmatrix}$

a) $A(B+C) = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 4 & 9 \end{pmatrix}$ $\underbrace{2\times2 \cdot 2\times2}_{2\times2}$

$$\begin{pmatrix} 8+0 & 6+0 \\ 4+8 & 3+18 \end{pmatrix} = \begin{pmatrix} 8 & 6 \\ 12 & 21 \end{pmatrix}$$

b) $A(B-C) = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} -6 & 1 \\ 2 & -5 \end{pmatrix}$

$$\begin{pmatrix} -12+0 & 2+0 \\ -6+4 & 1-10 \end{pmatrix} = \begin{pmatrix} -12 & 2 \\ -2 & -9 \end{pmatrix}$$

c) $(B+C) \cdot A = \begin{pmatrix} 4 & 3 \\ 4 & 9 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$

$$\begin{pmatrix} 8+3 & 0+6 \\ 8+9 & 0+18 \end{pmatrix} = \begin{pmatrix} 11 & 6 \\ 17 & 18 \end{pmatrix}$$

d) $A.B.C = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} -1 & 2 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 5 & 1 \\ 1 & 7 \end{pmatrix}$

$$\begin{pmatrix} -2+0 & 4+0 \\ -1+6 & 2+4 \end{pmatrix}\begin{pmatrix} 5 & 1 \\ 1 & 7 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 4 \\ 5 & 6 \end{pmatrix}\begin{pmatrix} 5 & 1 \\ 1 & 7 \end{pmatrix} = \begin{pmatrix} -10+4 & -2+28 \\ 25+6 & 5+42 \end{pmatrix} = \begin{pmatrix} -6 & 26 \\ 31 & 47 \end{pmatrix}$$

14.2.22 $M = \begin{bmatrix} -1 & 5 & 5 \\ 3 & -1 & 5 \\ 3 & 3 & -1 \end{bmatrix}$ $M^t = \begin{bmatrix} -1 & 3 & 3 \\ 5 & -1 & 3 \\ 5 & 5 & -1 \end{bmatrix}$ $\underbrace{3\times3 \cdot 3\times3}_{3\times3}$

$M^t \cdot M$

$$\begin{bmatrix} 1+9+9 & -5-3+9 & -5+15-3 \\ -5-3+9 & 25+1+9 & 25-5+3 \\ -5+15-3 & 25-5-3 & 25+25+1 \end{bmatrix} = \begin{bmatrix} 19 & 1 & 7 \\ 1 & 35 & 17 \\ 7 & 17 & 51 \end{bmatrix}$$

14.2.23 $A = \frac{1}{2}\begin{pmatrix} 8 & 5 \\ 6 & 5 \end{pmatrix}$ $B = \frac{1}{5}\begin{pmatrix} 5 & -5 \\ -6 & 8 \end{pmatrix}$ $\therefore AB = I$

$$\begin{pmatrix} 4 & 5/2 \\ 3 & 5/2 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -6/5 & 8/5 \end{pmatrix} = \begin{pmatrix} 4-3 & -4+4 \\ 3-3 & -3+4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

Lista de Exercícios – PDI

Bruna Virginia Neves

14.2.1 $a = (1,3,-1,5)$ $b = (4,4,2,5)$ $c = (3,1,3,0)$

a) $a+b+c = (8,8,4,10)$ b) $a+b-c = (2,6,-2,10)$

c) $a-b+c = (0,0,0,0)$ d) $a-b-c = (-6,-2,-6,0)$

14.2.2 If. $u = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $v = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$, $w = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$,

$$3u - v + w = \begin{pmatrix} 3x_1 \\ 3x_2 \end{pmatrix} - \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

$$= \begin{pmatrix} 3x_1 - y_1 \\ 3x_2 - y_2 \end{pmatrix} + \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} 3x_1 - y_1 + z_1 \\ 3x_2 - y_2 + z_2 \end{pmatrix}$$

14.2.3 $A = \begin{pmatrix} 3 & -2 & 1 \\ 0 & 5 & 4 \end{pmatrix}$ $B = \begin{pmatrix} -2 & 2 & 0 \\ 5 & 1 & 1 \end{pmatrix}$

a) $A + B = \begin{pmatrix} 1 & 0 & 1 \\ 5 & 6 & 5 \end{pmatrix}$ b) $A - B = \begin{pmatrix} 5 & -4 & 1 \\ -5 & 4 & 3 \end{pmatrix}$

c) $A + 2B = \begin{pmatrix} -1 & 2 & 1 \\ 10 & 7 & 6 \end{pmatrix}$ d) $3A - B = \begin{pmatrix} 11 & -8 & 3 \\ 5 & 14 & 11 \end{pmatrix}$

14.2.7

$f = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$ $g = \begin{pmatrix} 23 \\ 17 \\ 8 \\ 6 \\ 8 \\ 2 \end{pmatrix}$ $f \cdot g = \begin{pmatrix} 0 \\ 17 \\ 16 \\ 18 \\ 32 \\ 10 \end{pmatrix}$ $T = 93$ fams

14.2.16

$$\overset{A}{(x \ y \ 1)} \overset{B}{\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 4 \end{bmatrix}} \overset{C}{\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}}$$

$BC = \begin{bmatrix} 3x + y \\ x + 2y - 1 \\ -y + 4 \end{bmatrix}$ $1\times3,\ 3\times1 \to 1\times1$

AB: $1\times3,\ 3\times3 \to 1\times3$

$AB = [3x + y + 0 \quad x + 2y - 1 \quad -y + 4]$

$3\times3,\ 3\times1 \to 3\times1$ BC

$(AB)C = (3x^2 + xy + xy + 2y^2 - y - y + 4)$

$1\times3,\ 3\times1 \to 1\times1$ $= (3x^2 + 2y^2 + 2xy - 2y + 4)$

$\therefore (AB)C = A(BC)$

$A(BC) = 3x^2 + xy + xy + 2y^2 - y - y + 4$
$= 3x^2 + 2y^2 + 2xy - 2y + 4$

14.9.3 $\lambda = 1$

$$\begin{cases} x_1 = 0 \\ x_1 + 6x_2 + 3x_3 = 0 \\ 5x_1 + 4x_2 + 2x_3 = 0 \end{cases}$$

$$x_2 = \frac{-3x_3}{6} = \frac{-x_3}{2} \rightarrow x_2 = -\frac{x_3}{2}$$

$$x_2 = -\frac{2x_3}{4} \rightarrow -\frac{x_3}{2}$$

$$(x_1, x_2, x_3) = (0, -1/2\, x_3, x_3) = (0, -1/2, 1)x_3 \rightarrow (0, -1/2, 1)$$

$\lambda = 9$

$$\begin{cases} 7x_1 = 0 \\ x_1 - 2x_2 + 3x_3 = 0 \\ 5x_1 + 4x_2 - 6x_3 = 0 \end{cases}$$

$$x_3 = \frac{2x_2}{3}$$

$$x_3 = \frac{4x_2}{6} = \frac{2x_2}{3}$$

$$\left(0, x_2, \frac{2x_2}{3}\right) = (0, 1, 2/3)x_2$$

$$\left(0, 1, \frac{2}{3}\right)$$

$\therefore$

$\lambda_1 = 2 \qquad V_1 = (1/3, -2/3, 1)$

$\lambda_2 = 1 \qquad V_2 = (0, -1/2, 1)$

$\lambda_3 = 9 \qquad V_3 = (0, 1, 2/3)$