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Fundamental quantitative methods of land surface analysis

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Abstract

Effective quantitative land surface analyses in soil science need scale-free land surface attributes (morphometric variables, MVs) to be introduced for making comparable results obtained at different scales. To investigate the problem in more detail, a conceptual scheme and curvatures studied earlier in Shary (1995) [Math. Geol. 27 (1995) 373] are further developed in this paper, formulae for a complete system of 12 curvatures and some other MVs are given, and modified Evans–Young algorithm for curvature calculation is described that does not emphasize grid directions. The conceptual scheme is based on that MVs often describe not the land surface itself, but rather the system "land surface + vector field", where vector fields of common interest are gravitational field and solar irradiation. Correspondingly, morphometric variables and concepts may (1) refer to this system description (field-specific), or (2) be invariant with respect to any vector field (field-invariant), that is, describing the land surface itself, its geometrical form. From the other side, MVs and concepts may be (1) local, (2) regional, which need extended portions of a restricted part of land surface for their determination at a given point, or (3) global (planetary), when elevations of all the Earth are needed for their determination at a given point. Global MVs do not consist subject of this paper; so, the four classes of MVs are considered here: class A (local field-specific MVs), class B (regional field-specific), class C (local field-invariant), and class D (regional field-invariant). MVs of these classes permit description of geometrical land form, pre-requisits of surface runoff, thermal regime of slopes, and altitude zonality. Class A contains three independent MVs expressed by first derivatives of elevation Z by plan coordinates (slope steepness, slope direction, solar insolation) and seven curvatures expressed by second derivatives of Z; class C contains five curvatures; class B contains two variables (catchment and dispersal areas); MVs of class D are not introduced yet. Also,

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some non-system MVs of class A are described, sense of all MVs is described, and interrelationships between MVs are shown. Three curvatures are independent, not two, as this is often implied. It is experimentally shown that average depth of a depression defined in class B may not depend upon scale, while local MVs may not have limit values for large scales. Scale-free morphometric variables are defined here as those that have limit values for large scales. It is experimentally shown that maximal catchment area (class B) is a scale-free variable for thalwegs. These results show that local MVs are scale-specific (except elevation), but scale-free regional MVs might be introduced as a generalization of curvature concept. Two surface runoff accumulation mechanisms are considered in their relation to local and regional field-specific MVs; although the first one is generalized to a regional MV (catchment area), there is no regional MV for the second one description, although it is of great importance in soil science as describing slow profile changes. Geometrical forms were little studied in soil science; arguments are given that they may be useful for studying memory in soils, which is determined by temporal shifts between land surface formation and soil formation processes. The following topics are discussed: the current state of morphometry, an ambiguity in land form definitions, and a possibility to generalize curvature concept for regional scale-free MVs. The consideration is restricted by methods of the general geomorphometry; partial approaches are considered only by selection.

Keywords: Digital elevation model; Topography; Curvature; Scale; Invariant; Runoff

1. Introduction

Difficulties with detailed soil property description for sufficiently large terrains often exceed the real possibilities of measurements (McKenzie and Ryan, 1999). For this reason, and to study soil formation causes, it appears useful to use other environmental features (Odeh et al., 1991), land surface attributes belonging to the mostly important ones (Odeh et al., 1992a). For sufficiently strong correlation between soil and environmental features, soils might be predicted directly from environment (Webster, 1977). McKenzie and Ryan (1999) noticed in this relation that suitable environmental features are of more importance than the choice of statistical methods.

Quantitative analysis of relationships between land surface and soil variables has been studied by many authors. Land surface variables were studied, for example, in relation to soil water content and other hydrological features (Troeh, 1964; Beven and Kirkby, 1979; Sinai et al., 1981; Moore et al., 1988, 1993; Kuryakova et al., 1992; Bell et al., 1992, 1994; Fiez et al., 1995; Huang, 1995; Tomer and Anderson, 1995; McKenzie and Ryan, 1999), particle-size distribution in soils (Klich et al., 1990; Martz and de Jong, 1990; Odeh et al., 1991; Tomer and Anderson, 1995), soil horizon depths (Martz and de Jong, 1990; Pennock and de Jong, 1990; Carter and Ciolkosz, 1991; Tomer and Anderson, 1995; King et al., 1999; Sinowski and Auerswald, 1999), vegetation features (Sinai et al., 1981; Simmons et al., 1989; Fiez et al., 1995; Timlin et al., 1998). Some authors attempted to classify soils and landscapes on such a basis (Troeh, 1964; Pennock and de Jong, 1990; Odeh et al., 1992a,b; Moore et al., 1993; Stepanov, 1996; McKenzie and Ryan, 1999).

However, these studies have revealed some essential problems in this direction. First, most of currently known variables for land surface description are scale-specific, making difficult comparisons of results obtained at different scales. Second, non-classificational nature of current land surface analysis methods has resulted in some missing or not sufficiently recognized ideas, such as those of geometrical form description, which are known in other sciences (e.g., in physics and mathematics) but were studied very little in soil science. Third, the system of land surface attributes seems to be not standardized and a large diversity of non-strictly defined and often interdependent land surface variables, frequently insufficiently described, seriously hinders comparison of results from different authors. This encourages the development of a classificational system of concepts of morphometry, the standardization of a set of morphometric variables, the elucidation of relationships between them, and conditions for their use in soil and landscape studies for comparable results. Bearing in mind these purposes, we have restricted consideration in this paper to the variables and concepts that are based on several invariant properties of land surface attributes.

Methods of morphometry can be used to analyse the following:

(i) geometrical land surface forms,
(ii) morphometric pre-requisits of surface runoff of dry and liquid materials,
(iii) morphometric pre-requisits of thermal regime of slopes,
(iv) altitude zonality as a pre-requisite of atmospheric temperature and pressure changes.

In case (i), the land surface itself is a subject of analysis; in cases (ii,iv) and (iii), the systems "land surface + gravitational field" and "land surface + solar irradiation" are analysed, respectively. More generally, the system "land surface + vector field" is analysed in the last three cases. Although the surface itself was a subject of special attention in physics and mathematics during several centuries, the system "surface + vector field" as a mathematically double object was not sufficiently studied (see below), and for the real land surface (which is not smooth, as substantiated in the Section 3 below), it needs special knowledge on the maximum precision of analysis achievable. Pre-requisits of processes for the system "land surface + vector field" imply that each kind of vector field acts like a tendency, not completely defining respective processes. In gravitational field, corresponding land surface attributes are pre-requisits of material movement that do not completely define the results (i.e., material reposition) with a tendency of results to appear later than pre-requisits (in land surface formation), after some temporal shift. This concept was considered in more detail by Stepanov et al. (1991). Wind, snow melting, evapotranspiration, diffusion, inertia (e.g., for water streams), human activities are commonly known examples of other pre-requisits.

1.1. General geomorphometry

For further considerations, some points of studies on land surface features may be represented as the following four directions.

(1) Gauss (1827) created the basis of modern differential geometry of surfaces, which is a theory of surface geometrical forms described locally, that is, by curvatures. Applications of this approach were developed, for example, in the physics of capillary phenomena (Finn, 1986).

(2) Maxwell (1870) tried to develop a quite different approach, considering simultaneously the land surface for the whole Earth and its gravitational field. This is the theory of the system "land surface + gravitational field", considered globally, for the entire planet. However, variables for basin description have no sense without taking into account land surface boundaries that are absent for a whole planet, in contrast to a part of it. The first ideas of restricted land surface portion analysis were suggested by Speight (1968, 1974) in his studies of the system "land surface + gravitational field" for such portions, which we term here "regions".

(3) Evans (1972) and Krcho (1973) studied morphometric variables (local, such as plan and profile curvatures) that are absent in the theory of Gauss (1827). This is because Gauss studied the geometrical form of surfaces, while the listed geomorphologists studied features of the system "land surface + gravitational field" (or field-specific forms).

(4) In Earth sciences, to our knowledge, nobody has studied geometrical land surface forms for a whole planet; the corresponding direction in mathematics is known as topology. Matheron (1965, 1975) used for regions "regionalized variables", which are better known in geostatistics, than in morphometry.

Evans has attributed the approaches under consideration to a discipline that he termed general geomorphometry. He wrote (Evans, 1972): "General geomorphometry as a whole provides a basis for the quantitative comparison even of qualitatively different landscapes, and it can adapt methods of surface analysis used outside geomorphology. Specific geomorphometry is more limited; it involves more arbitrary decisions, and leaves more room for subjectivity in the quantification of its concepts." Nevertheless, in spite of some attempts to integrate approaches known in physics and mathematics (e.g., Koenderink and van Doorn, 1994), general geomorphometry has remained a non-systematical set of methods, which is developing mostly as a continuation of the ideas of Evans and Speight, but does not use the achievements of Gauss; the problem of scale-free morphometric variables also remains at its beginning.

The general physico-mathematical theory of this discipline received further development in the works of Shary (1991, 1995), where an integrating approach was outlined with the purpose of joining and systematizing the directions listed above. In Shary (1995), a classification of surface points by signs of curvatures was constructed, and its completeness was proved.

In this paper, we describe this system of methods together with their formulae, develop the system of concepts of general geomorphometry as a science, and analyse questions of maximally possible precision and the possible existence of limits for morphometric variables at large scales. A special attention is paid to various invariants, and to the problem of how and where (in which class) might scale-free variables be introduced.

1.2. Coordinate system invariants

Three kinds of variables might be considered, which describe land surface and are coordinate system invariants: those with respect to surface shift, to surface inversion (i.e., replacing elevation Z by $-Z$), and to surface rotation. Some statistical results based on the first two were studied by Shary (1995); consider now the third kind of variables. The following three types of such variables are considered here as being of special value:

(i) variables that are invariant with respect to surface rotation around any axis (for geometrical form description);
(ii) isotropic variables that are invariant with respect to surface rotation around vertical axis (e.g., for surface runoff description);
(iii) not invariant anisotropic variables that depend on some geographical direction (e.g., slope aspect or slope insolation).

Variables (i) that lie in the basis of the differential geometry of surfaces describe what is termed geometrical forms here; such variables are field-invariant because they do not depend on a specific field direction. Since topographic maps and digital elevation models (DEMs) are always given in coordinate systems, where one of axes is vertical (i.e., coordinates are linked to the gravitational field), most variables for surface runoff description are isotropic. Variables for solar irradiation are usually anisotropic (except for the Sun position in zenith, when one of axes direction and solar irradiation vector field coincide). Considering variables (ii) for surface runoff description, Martz and de Jong (1988) noticed that land surface attribute usually "is described in the context of some process or flow to be taking place within the landscape"; this is not necessary for geometrical form attributes introduction because no surface runoff is considered. As Gauss (1827) showed, special efforts are needed to make variables field-invariant; because of this, numerous land form attributes studied in Earth sciences appeared not field-invariant and do not describe geometrical form. The concept of geometrical and field-specific land form is explained in more detail in the following section.

Correspondingly, geometrical form attributes (variables of classes C and D in the consideration below) were studied very little in soil science. In some studies (e.g., Sinai et al., 1981; Timlin et al., 1998), soil and vegetation properties were compared with elevation Laplacian, approximately equal to minus double mean curvature (as shown in Section 5 below), which is one of important characteristics of geometrical form; similar comparisons with mean curvature were reported by Kuryakova et al. (1992). Nevertheless, no explanations of unexpectedly high correlations and no understanding of this variable were reported.

1.3. Principles of a general classification

The land surface exists in several vector fields — solar irradiation, gravitational, magnetic, electrical, etc. The most important in practice is the gravitational field, since it controls the formation of surface runoff of dry and liquid materials, and corresponding level sets of the system "land surface – gravitational field" (contour lines) are plotted on topographic maps, providing giant sets of data, including those available as DEMs. For this reason, all morphometric variables can be subdivided into two groups, namely variables characterizing land surface without taking the vector field into account, and variables that take it into account. In this relation, Shary (1995) has suggested subdivision all of morphometric variables and related concepts by their relation to the gravitational or

Table 1
Classes of morphometric variables and concepts

Morphometric variables and concepts	Local	Regional
Field-specific	Class A	Class B
Field-invariant	Class C	Class D

other field, also taking into account their local or non-local nature. Such classification for regions is shown in Table 1. This classification corresponds (except for planetary aspects) to the four directions, listed above.

To carry out calculations and morphometric maps construction in this paper, a Geographical Informational System (GIS) "ECO" for IBM PCs was used, developed by one of the authors.

2. The four classes of morphometric variables and concepts

The classification of Table 1 is based on morphometric variables (MVs) that are defined in each surface point. Morphometric characteristics (such as contour area, or an MV) that do not necessarily refer to some surface point are considered as more general concept. According to the above consideration, also global (planetary) classes B′ and D′ can be introduced for global field-specific (class B′) and global field-invariant (class D′) MVs in the general classification that can be defined only for a whole planet, not for a region. MVs of these global classes will not be considered in this paper.

2.1. Geometrical and field-specific land forms

Concepts and variables of classes C and D characterize geometrical forms of a land surface that are invariant with respect to any coordinate system rotation. In this case, the form of a body is understood as the relative position of points on the surface that bounds it, and is defined by distances between these points. A depression of the class D, which is not able to hold water (Fig. 1a), is an example of a geometrical form. From a geometrical point of view, concepts of the classes A and B describe forms that arise on the land surface by means of a vector field (that are termed here field-specific forms), such as a B-depression (Fig. 1b) defined by its ability to hold water. Clearly, it is impossible to give a definition of a B-depression (i.e., a depression defined in the class B) without taking Earth gravitational field into account. A shadow from a hill is a commonly known example of a field-specific form for the system "land surface + solar irradiation". From the other side, as mentioned in the Introduction, in the gravitational field MVs and concepts of classes A and B describe morphometric pre-requisits of land surface material runoff, and in solar irradiation—those of the thermal regime of slopes.

2.2. Local and regional morphometric variables

At a given point, local MVs are defined by consideration of a fixed size (not defined by terrain specifics) portion of the terrain immediately surrounding that point,

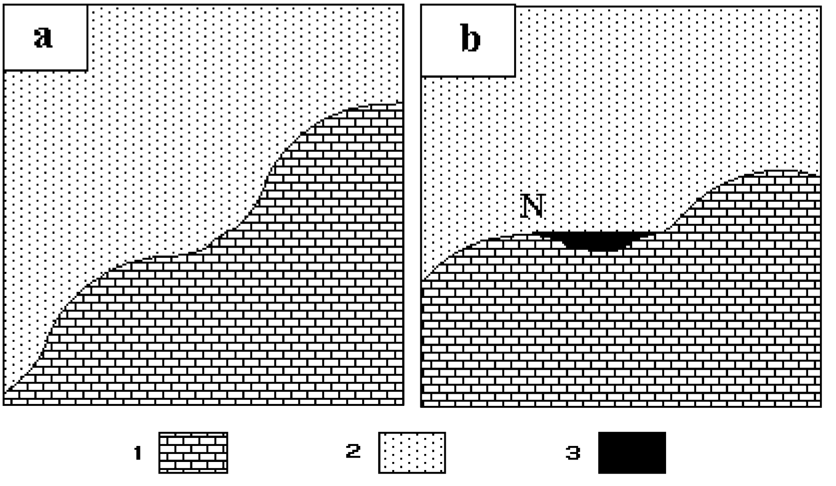


Fig. 1. On the concepts of D- and B-depressions (i.e., depressions of classes D and B). This is an example of two surfaces equal by form, but differently oriented in the gravitational field. 1— rocks, 2— air, 3 — water. The left is a D-depression (a broadly concave piece of land surface, it may have locally convex parts), right is a B-depression (a part of broadly concave land surface piece that is able to hold water).

in contrast to regional MVs, for which this portion principally depends upon the terrain specifics.

In other words, filters of a fixed size (convenient through their simplicity) are used in local morphometric analysis, and they delineate land surface forms. For example, at the grid mesh 10 m, forms of larger sizes are delineated, information on forms of smaller sizes is lost. Information on larger land forms in this case may be revealed by visual observation of maps, for example, a group of hills detected at some given filter size may be associated by an observer with a ridge of much larger size, in comparison with the grid mesh.

With regional MVs, a principle of "searching" filter is used; filter size is not specified in advance, but is determined by terrain specifics (for example, by a B-depression size), or by map boundaries. In the case of global variables (and also of land surface elevation), the concept of precision of some land surface form determination may have the same sense, as in carrying out a geodetic survey. For example, a B-depression's boundary is the largest closed contour line, precision in determination of which coincides with that of elevations; the situation is similar in the case of a D-depression, the boundary of which, however, does not coincide with a contour line (Fig. 1a).

Consider B-depression depth as an example of a regional MV. This depth is calculated from the lowest outlet elevation of the B-depression (Fig. 1b, point N) and can be chosen equal to zero outside B-depressions. For determination of this MV at a point M, it is not enough to consider its restricted vicinity, because even if in this vicinity of M a hill is detected, this does not mean that this hill is not on the bottom of the B-depression (for example, a hill at the bottom of a foundation pit). This is implied by the regional feature of the considered variable.

In a more strict understanding, when one takes into account a band of land surface that borders the considered part of terrain Ω, this does not change values of local variables in points of Ω. In contrast to this, taking this border into account may essentially change values of regional MVs in points of Ω. For example, if elevation values of the border are sufficiently large, taking it into account will result in increase of B-depression depths in all points of Ω.

At this time, a system of dependent from first and second derivatives variables of the general geomorphometry is introduced for classes A and C, the completeness of which was substantiated by Shary (1995), two variables are known for class B (see below), and no variables are known for class D.

2.3. Discrete forms

It is not too difficult on the basis of the B-depression concept, for which algorithm of calculation for the regional case was described by Martz and de Jong (1988), to introduce the concept and algorithm for a B-hill. The latter can be obtained by the inversion of an elevation matrix (replacing Z by $-Z$), after which for calculation of B-hills, the same algorithm can be used, as for B-depressions.

Consider a copy of land surface in which all B-depressions were filled by water, and after that, this water has been frozen. One may take these pieces of ice as discrete entities (separately one from another), measure their volumes, sizes, and so on. For this reason, we term such land forms discrete forms of the land surface; they have precisely determined characteristics (see below) and have a volume. Another example of a discrete form (for the system "land surface + solar irradiation") is a shadow from a hill during a sunny day. Although shadows change relatively fast, any land form is changing, and we cannot see reasons why geometry (geomorphometry) cannot be applied to instantaneous form of shadows.

2.4. Boundary effects

When DEMs are available only for a part of B-depression (e.g., for a half of lake), it is impossible to calculate correct depths of that depression (because a half of lake is not a lake). Boundary effects are of special importance for regional MVs. For example, when an up-slope part of a basin is outside given DEMs, MVs that describe the basin (such as maximal catchment area, see below) may have incorrect values. In this case, one should use additional DEMs to improve the situation (in the example, DEMs which include all the up-slope part of the basin); in general, regional MVs use needs some preliminary evaluation of DEMs for a particular purpose of investigation.

2.5. Scale-specific and scale-free variables

MVs that do not depend on a particular scale (at least, for sufficiently large scales), are termed 'scale-free' here, other MVs being scale-specific. Scale-free MVs are of great importance in soil science because they provide a way to make results of terrain analyses comparable. Clearly for very small scales (say, only several contour lines for all the Earth), any MV will be scale-specific; so, it is expected that, when some MV appear scale-free, this means that such MV has a limit value in each land surface point for large scales. An essential question considered below in this paper is: where (in which class) and how might such scale-free MVs be introduced?

3. The basic problem in geomorphometry

Data sources for calculation of maps of MVs are usually DEMs in the form of elevation matrices (e.g., Evans, 1972; Tomer and Anderson, 1995; McKenzie and Ryan, 1999). A grid mesh is explicitly defined in an elevation matrix, on the value of which some MV at a given point may depend. It was experimentally and theoretically shown (Mandelbrot, 1967; Clarke, 1988) that the model of a smooth land surface is not in agreement with observation. For example, the existence of contour line length that follows from the smooth model is in contradiction with L.F. Richardson's measurements (see Mandelbrot, 1967). For this reason, the land surface cannot be regarded as smooth, and therefore, MVs obtained by differentiation have no objective values without relation to grid mesh or scale because a non-smooth surface has no derivatives.

This phenomenon results in dependence of such MVs on grid mesh. Evans (1975), who studied methods of the class A, has characterized the situation in the following way: "A basic problem in geomorphometry is that any measurement varies with scale, i.e. with the extent of the area or line involved." However, the elevation (at a given point, not average for some area) forms an obvious exception from this non-obvious rule. In addition, in contrast to this common belief, the situation does not seem so hopeless for regional concepts. Here, we demonstrate this experimentally for a B-depression (below also an example is given for an MV of class B). Compare now the dependence of B-depression average depth on the grid mesh w with the similar dependence for average slope gradient (Fig. 2). The increase of grid mesh was achieved by means of matrix thinning using a technique, similar to that described in Evans (1995), the size of the initial matrix (with grid mesh 20 m) being 1065 by 510 elements. To exclude boundary effects, the studied matrix was taken as a part of a larger one, with a reserve of rows and columns on each side (this reserve was also used in calculations for Fig. 3).

This experiment (and also that given below) shows that, at least for one characteristic of class B, the basic problem in geomorphometry is not so insurmountable. Essentially, this problem is that of presence or absence of a limit value for a given morphometric characteristic as $w \to 0$. For B-depression average depth and also for some another characteristics (e.g., for maximal depth and area), such limit values do exist because these characteristics are expressed through precisely defined elevations and need no differentiation of a non-smooth surface. It follows from land surface continuity that a difference between this limit value and real one (for given w) is very small for sufficiently small w, and may appear not valid for some terrains, for which land surface cannot be considered as continuous (e.g., in mountainous terrains with essential continuity breaks). Thus, to find close to limit characteristics of a B-

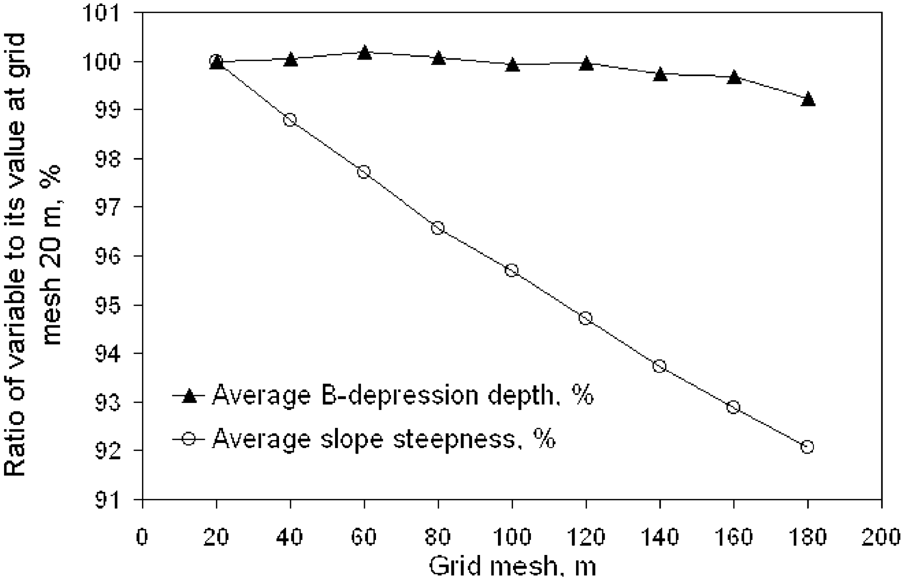


Fig. 2. The dependence of average depth of the lake Königssee in Germany and average slope steepness for the same terrain on grid mesh w. The scale of initial topographic map is 1:50,000 and it contains data on the lake bathymetry. For both the variables, their ratios (in %) to their initial value at grid mesh 20 m (6.23 m and 17.4°, respectively) are shown. It is seen that the average depth of the lake practically does not depend on w, while the average slope steepness essentially decreases with w.

depression, one needs to choose sufficiently small grid mesh (or sufficiently large scale). It will now be demonstrated that for average slope gradient a corresponding finite limit may be absent.

If elevation does not depend upon the plan coordinate y, when determined at grid mesh w slope gradient is $\Delta z/w$, where Δz is the corresponding change in elevation. The average slope gradient is Z/w, where Z is average value of Δz_i (the index i counts matrix elements). If the trend of a quasi-stochastic, "noisy" surface is a horizontal plane, all Δz_i may have the same order of value and Z may not depend on w. In this case, the slope gradient is proportional to $1/w$, that is, it infinitely increases as $w \to 0$, and tends to 0 as $w \to \infty$; the logarithm of average slope gradient as a function of w will be a straight line with a negative slope in this case. The quasi-stochastic surface can be modelled by adding pseudorandom numbers to a horizontal plane and the average slope gradient can be calculated for a matrix with a grid mesh w by the Evans–Young method (Pennock et al., 1987) for grid meshes $w, 2w, \ldots, nw$. The experimental results are shown in Fig. 3. The dependence of average slope gradient on grid mesh, first noted by Evans (1975), is confirmed, with some addition on its origin.

The linear dependence of the average slope gradient on w for the quasi-stochastic surface with a horizontal trend in logarithmic coordinates shows that, at least for this case, average slope gradient may be considered as tending to infinity as $w \to 0$. Dependencies

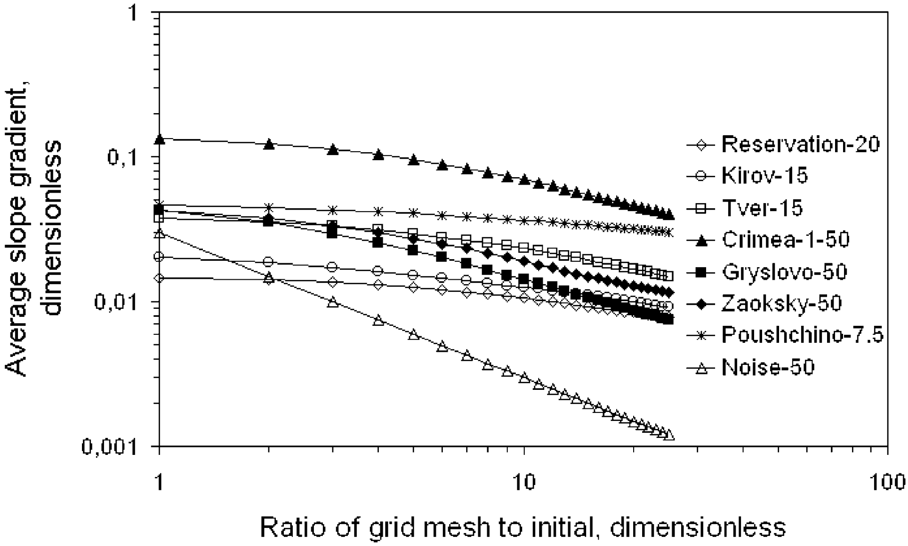


Fig. 3. The dependence of average slope gradient on grid mesh w. Results are shown for seven terrains and for a quasi-stochastic surface with "noise" amplitude 10 m (denoted as Noise-50), double logarithmic coordinates. Size of each matrix was 300 by 300 elements (a number after terrain abbreviation means grid mesh in meters). Grid mesh used for calculations is expressed as ratio to initial matrix grid mesh.

for real terrains differ from linear; nevertheless, the average slope gradient does not demonstrate a tendency to tend to some finite limit as $w \to 0$.

4. Modified Evans–Young method

The basis of the Evans–Young method is that for derivative calculation using an elevation matrix "quadratic trend surfaces should be fitted to the neighbourhood (3×3) of each sample point" (Evans, 1972). Calculation formulae for this algorithm were suggested by Young (1978) in a joint investigation with Evans (1979). The method was later described in Geoderma (Pennock et al., 1987) with a reference to these studies.

The essence of the Evans–Young method consists in the following. The non-smooth land surface in a vicinity of a given point is replaced by a "sheet" of a smooth surface (the second-order polynomial) that is fitted to the real surface by a least squares criterion. After that, this smooth surface ("sheet") is used for calculations of MVs that are expressed by first and second partial derivatives.

In the original variant, the Evans–Young method provides good results for first derivative maps, but essentially emphasizes grid directions for second derivatives (i.e., for land surface curvatures), as can be seen on maps of these MVs (e.g., Evans, 1979, 1980, 1990). Although a set of finite difference methods have been suggested later (see Evans and Cox, 1999), there is no commonly used solution for the question of Evans–

Young method modification. A modification of this method, developed by P.A. Shary, is now suggested.

In the first phase of calculation, points of a subgrid 5×5 are used for parametric (weak) isotropic surface smoothing of the nine internal points of the subgrid. This smoothing complies with the following criteria: (i) a plane piece of surface is transformed into a plane one, (ii) the smoothing filter is isotropic, (iii) the filter weights decrease linearly with the distance from the center of the subgrid. In the second phase, the original Evans–Young method is used for the smoothed subgrid 3×3. As a result, grid directions on maps disappear (see example of map images below). Deduction and formulae of the modified Evans–Young method are given in the Appendix A.

This method was successfully (with no essential grid directions visible) used with DEMs at grid meshes from 1 m to 2.5 km containing in a whole about 5 millions of elements. When some DEM is interpolated to a smaller grid mesh, say from 20 to 1 m, grid directions visibility depends on an interpolation process used, but such interpolated data does not contain information on these small land forms and curvature maps may reflect details not found in the original DEM. Another item is that for too large grid meshes (>1 km), no extended, multi-pixel structures may appear for some curvatures, such as spurs of hills given by horizontal curvature (see below), because these hills may appear of the same order in their plan size.

5. The system of morphometric variables

The system of MVs under consideration in this paper includes: (1) elevation (used as initial data for calculation of other MVs), class A; (2) MVs expressed by first derivatives, class A; (3) MVs expressed by second derivatives, including the complete system of land surface curvatures in classes A and C; (4) contour line and flow line curvatures that are considered as being outside the complete system of curvatures, partially repeating properties of MVs of the latter; (5) the two currently known regional MVs: maximal catchment area and maximal dispersal area, class B; and also (6) described below in this paper hydrodynamic area and daily solar radiation dose, class B. To our knowledge, the described set of MVs for geometrical land form description (class C) has not been used in soil science in any systematical way.

5.1. First derivatives

Slope steepness (GA) is a quantitative measure of maximal rate of elevation change in gravitational field; it ranges from 0 to 90°. Geometrically, it is an angle between horizontal plane and tangential to the surface plane. It is calculated using a commonly known formula (e.g., Krcho, 1973), which represents this angle through first partial derivatives (Table 2). The slope gradient G is tan(GA); it ranges from 0 to infinity. Some authors (e.g., Moore et al., 1993; McKenzie and Ryan, 1999) use $\mathrm{GP} = 100G\%$, which equals 100% for $\mathrm{GA} = 45°$ and have the same range as G. Runoff on the land surface occurs under tangential to the land surface component of the gravitational force. It is known (e.g., Strahler, 1952) that this component is proportional not to the slope steepness or gradient, but to the gradient

Table 2
Formulae for calculation of local morphometric variables; in these formulae, $p=\partial z/\partial x$; $q=\partial z/\partial y$; $r=\partial^2 z/\partial x^2$, $s=\partial^2 z/\partial x \partial y$; $t=\partial^2 z/\partial y^2$ are designations for partial derivatives of the surface $z=z(x,y)$; the function sign(x) equals 1 for $x>0$, 0 for $x=0$, and -1 for $x<0$

Variable name	Class	Unit	Formula	Reference
Slope steepness	A	°	$\mathrm{GA}=\arctan(p^2+q^2)^{1/2}$	–
Slope gradient	A	–	$G=(p^2+q^2)^{1/2}$	–
Gradient factor	A	–	$\mathrm{GF}=(p^2+q^2)^{1/2}/[1+p^2+q^2]^{1/2}$	–
Slope direction (or aspect)	A	°	$A_0=-90[1-\mathrm{sign}(q)](1-\lvert\mathrm{sign}(p)\rvert)+180[1+\mathrm{sign}(p)]-180\mathrm{sign}(p)\arccos[-q/(p^2+q^2)^{1/2}]/\pi$	–
Mean curvature	C	1/m	$H=-[(1+q^2)r-2pqs+(1+p^2)t]/[2(1+p^2+q^2)^{3/2}]$	(Gauss, 1827)
Unsphericity	C	1/m	$M=[\{r[(1+q^2)/(1+p^2)]^{1/2}-t/[(1+q^2)/(1+p^2)]^{1/2}\}^2/(1+p^2+q^2)+\{pqr[(1+q^2)/(1+p^2)]^{1/2}-2[(1+q^2)(1+p^2)]^{1/2}s+pqt/[(1+q^2)/(1+p^2)]^{1/2}\}^2]^{1/2}/[2(1+p^2+q^2)^{3/2}]$	(Shary, 1995)
Difference curvature	A	1/m	$E=(q^2r-2pqs+p^2t)/[(p^2+q^2)(1+p^2+q^2)^{1/2}]-[(1+q^2)r-2pqs+(1+p^2)t]/[2(1+p^2+q^2)^{3/2}]$	(Shary, 1995)
Horizontal curvature	A	1/m	$\mathrm{kh}=-(q^2r-2pqs+p^2t)/[(p^2+q^2)(1+p^2+q^2)^{1/2}]$	(Krcho, 1983)
Plan curvature[a]	A	1/m	$\mathrm{kp}=-(q^2r-2pqs+p^2t)/(p^2+q^2)^{3/2}$	(Evans, 1972)
Vertical (=profile) curvature	A	1/m	$\mathrm{kv}=-(p^2r+2pqs+q^2t)/[(p^2+q^2)(1+p^2+q^2)^{3/2}]$	(Evans, 1972)
Rotor[a]	A	1/m	$\mathrm{rot}=[(p^2-q^2)s-pq(r-t)]/(p^2+q^2)^{3/2}$	(Shary, 1991)
Horizontal excess curvature	A	1/m	$\mathrm{khe}=M-E$	(Shary, 1995)
Vertical excess curvature	A	1/m	$\mathrm{kve}=M+E$	(Shary, 1995)
Minimal curvature	C	1/m	$\mathrm{kmin}=H-M$	–
Maximal curvature	C	1/m	$\mathrm{kmax}=H+M$	–
Total Gaussian curvature	C	$1/\mathrm{m}^2$	$K=(rt-s^2)/(1+p^2+q^2)^2$	(Gauss, 1827)
Total ring curvature	A	$1/\mathrm{m}^2$	$\mathrm{KR}=[(p^2-q^2)s-pq(r-t)]^2/[(p^2+q^2)^2(1+p^2+q^2)^2]$	(Shary, 1995)
Total accumulation curvature	A	$1/\mathrm{m}^2$	$\mathrm{KA}=H^2-E^2$	(Shary, 1995)
Slope insolation[b]	A	%	$F=50\{1+\mathrm{sign}[\cos a-\sin a^*(p\sin b+q\cos b)]\}[\cos a-\sin a^*(p\sin b+q\cos b)]/(1+p^2+q^2)$	(e.g., Onorati et al., 1992)

[a] These curvatures do not belong to the complete system of 12 curvatures (Shary, 1995).
[b] Here $90°-a$ is Sun vertical angle from horizon, b is Sun azimuth counted from northern direction clockwise.

factor (sin(GA)), and the formula (Table 2) deduction for it can be found, for example, in Shary (1991).

Slope direction (or aspect) is measured from the North clockwise and is the azimuth of elevation gradient (Table 2); formula deduction for it can be found, for example, in Young (1978) and Pennock et al. (1987).

Slope gradient and slope direction are components of a single entity, namely of two-dimensional vector of elevation gradient (Evans, 1972). We imply this vector taken with a negative sign because for southern slope aspect, gradient vector is directed to the North (e.g., Shary, 1991). The direction of this vector is the slope direction, and the length of it is the slope gradient.

Elevation, slope steepness and slope direction are not sufficient for land surface description even in local classes A and C (Evans, 1972). The major reason is that land surface form description is carried out at the local level by means of surface curvatures (Shary, 1995). It has been proven also (Shary, 1995) that there are three independent curvatures, not two, as is often, implicitly or explicitly, suggested in the literature (e.g., Evans, 1972, 1979; Evans and Cox, 1999; Krcho, 1973, 1983; Pennock et al., 1987; Tomer and Anderson, 1995; Thomas et al., 1999), and that signs of at least five curvatures are necessary to take into account for classification of land surface forms (Shary, 1995, Theorem 2). On a conceptual level, this is because elevation and gradient do not describe the two major accumulation mechanisms, which govern surface runoff of dry and liquid materials. Consider now what are these mechanisms and how might they be described quantitatively.

5.2. Two accumulation mechanisms

The first accumulation mechanism reflects surface flows convergence (Fig. 4, left). For one part of the surface flow lines (shown by arrows in the figure) can converge, for others — diverge. For areas where flow lines are convergent, the term convergent areas is used in the literature, where they diverge—divergent areas (e.g., Evans, 1972; Shary, 1995).

It is known that divergence (div) is the quantitative measure of flow line divergence: where div is negative, flow lines converge, where positive—diverge. It was proven (Shary, 1995, Theorem 4) that div of flow lines equals the plan curvature of a surface, kp (Table 2), that is, a correspondingly signed contour line curvature; a formula for calculating kp can be found, for example, in Krcho (1973). This is the basis for quantitative description of the first accumulation mechanism.

Horizontal curvature (kh, Table 2) was first introduced by Krcho (1983); it was proven (Shary, 1995) that the signs of kp and kh coincide. For this reason, the first accumulation mechanism can be reflected by a map of kh (Fig. 5c). Plan curvature may not be sufficiently convenient, because it tends to infinity near top of a hill or bottom of a pit, creating some problems in its usage (Shary and Stepanov, 1991). The infinity arises because near a top of a hill, the radius R of a contour line circle tends to 0, and the contour line curvature is $1/R$. Unusual statistics for kp (presence of large values) were mentioned in Evans (1979) and Shary and Stepanov (1991). The plan curvature does not belong to the complete system of 12 curvatures described herein.

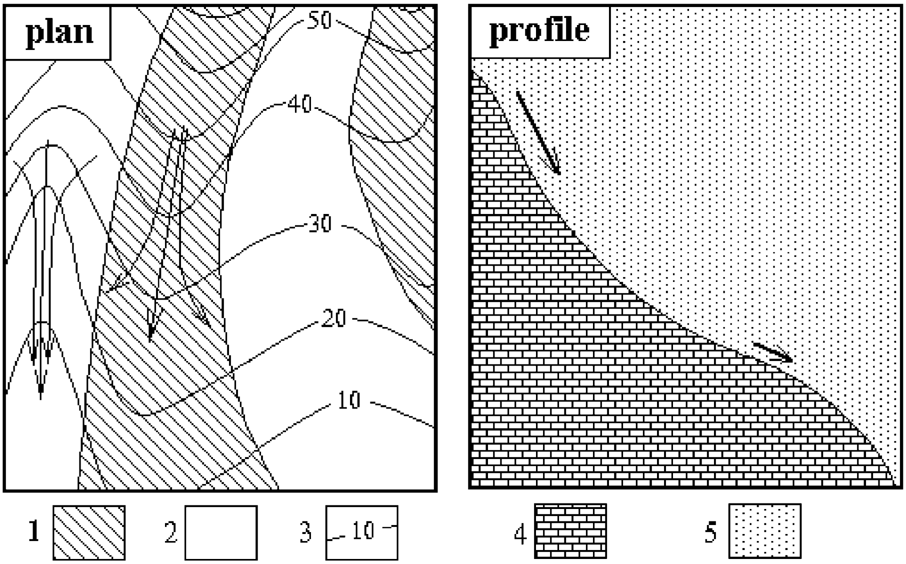


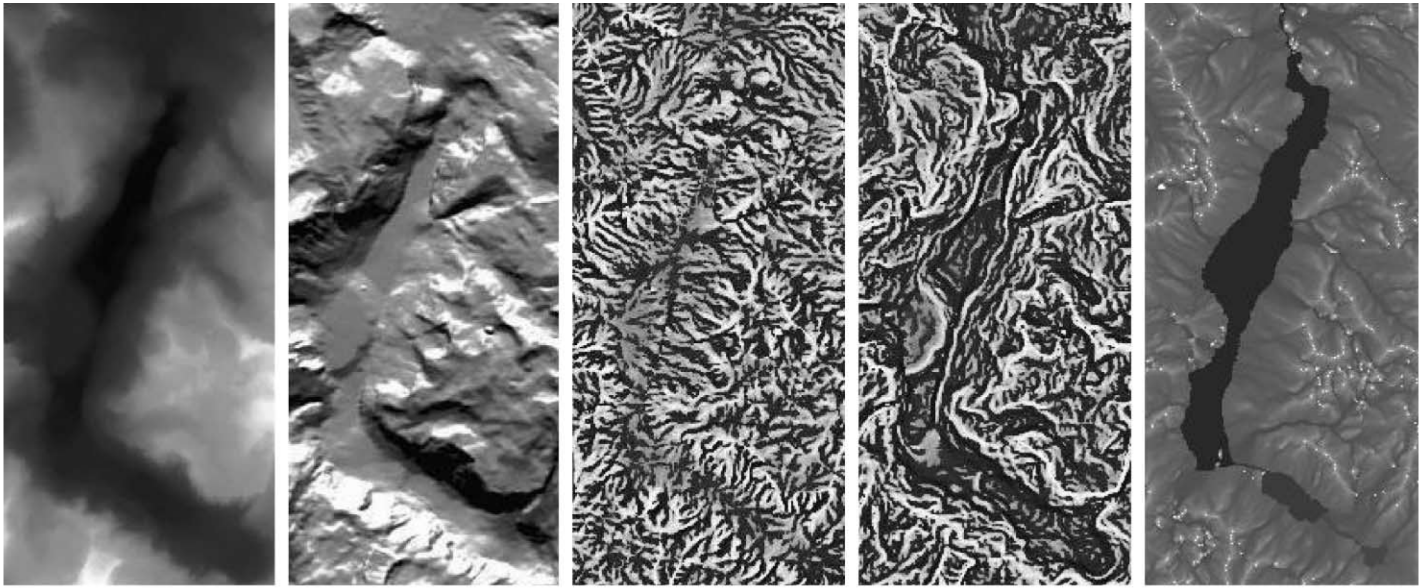
Fig. 4. Two accumulation mechanisms. Left — flow convergence/divergence, right — relative flow deceleration/acceleration. 1 — divergence areas, 2 — convergence areas, 3 — contour lines, 4 — rocks, 5 — air. See text for details.

The second accumulation mechanism reflects flows relative deceleration. On concave parts of a slope profile, the slope steepness diminishes downslope (Fig. 4, right—lengths of arrows are proportional to the slope steepness). Up-slope parts of profile are steeper; therefore, particles located there may move faster than those located below. As a result, material flows suffer relative deceleration on parts of the surface, and relative acceleration on others.

It was proven (Shary, 1995, Theorem 5) that the derivative of gradient factor GF by flow line length is the vertical (or profile) curvature kv (Table 2); deduction of a formula for kv can be found in Krcho (1973). There was an error in the statement of this theorem in Shary (1995); it follows from formula $\mathrm{kv} = (\mathrm{d}G/\mathrm{d}u)/(1 + G^2)^{3/2}$ proven in Shary (1991), where u is flow line length, and formula $\mathrm{GF} = G/(1 + G^2)^{1/2}$, which is seen from Table 2, that $\mathrm{kv} = \mathrm{d(GF)}/\mathrm{d}u$. This is the basis for quantitative description of the second accumulation mechanism. So, the second accumulation mechanism acts on profile-concave slopes, where $\mathrm{kv} < 0$ (Fig. 4, right), and can be described by a map of kv (Fig. 5d).

5.3. Four directions on a land surface

For consideration of the geometrical sense of curvature, the concept of a normal section of the surface is useful, which is known from the differential geometry of surfaces (e.g., Pogorelov, 1974). Let $\boldsymbol{n}$ be a normal vector (at the point $\boldsymbol{X}$) to the surface S (Fig. 6). A curve, which results from intersection of the surface S and some plane passing through the normal $\boldsymbol{n}$, is known as the normal section of the surface S at the point $\boldsymbol{X}$.



a b c d e

Fig. 5. Maps of some morphometric variables for mountainous terrain in Bavaria (surroundings of the lake Königssee), Germany. Grid mesh 60 m, the size of the terrain is 10.2×21.3 km. a — elevations (larger elevations are shown by lighter colors), b — insolation F (azimuth of the light source is 0°, angle from horizon is 35°; lighter colors refer to northern slopes, darker — to southern), c — horizontal curvature kh (light colors refer to positive kh, dark — to negative), d — vertical curvature kv (light colors refer to positive kv, dark — to negative), e — maximal catchment area MCA (darker colors refer to larger MCA, the largest dark area corresponding to the lake Königssee). Curvatures were calculated by the modified Evans–Young method, curvatures and MCA are logarithmed using formula (1).

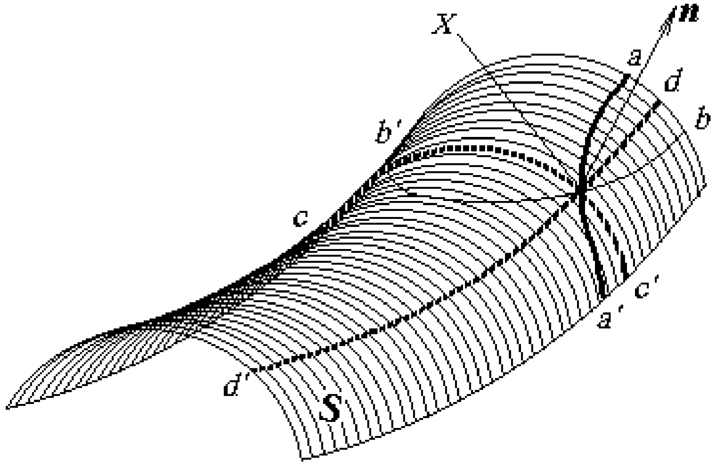


Fig. 6. The four directions naturally marked on the surface *S*. ***n***—normal vector to *S* at the point ***X***; aa′— gradient line; bb′— contour line; dd′, cc′— main normal sections.

The form of the surface itself marks two normal sections on it—that having a maximum of its curvature (Fig. 6, curve cc′) and that having a minimum (Fig. 6, curve dd′) among all possible normal sections at the point ***X***. Additionally, the gravitational field marks at the same point two other normal sections—tangential to the contour line (Fig. 6, curve bb′), and perpendicular to it, along which water flows (Fig. 6, curve aa′). Curvatures of the first two normal sections are known as maximal kmax and minimal kmin, correspondingly. Shary (1995) has proved that these curvatures may be calculated from other curvatures, as in Table 2. Curvatures of the two other normal sections of Fig. 6 are kv and kh.

The curvatures kmax and kmin can be used for construction of geometrical ridge and valley forms (Shary, 1995), which appear effective for decoding of lineaments, central type structures and so on in structural geology.

5.4. Other curvatures

Gauss (1827) proved that total Gaussian curvature $K = \mathrm{kmax} \cdot \mathrm{kmin}$ does not change for any surface bending that does not change lengths of curves on it (i.e., when the surface is not rumpled and not stretched). This theorem is widely used in geodesy and cartography, for example, in constructing projections of objects on the Earth on surfaces like cone, which can be unrolled onto a plane sheet without lengths changing. The formula for *K* in Table 2 was firstly deduced by Gauss (1827).

According to Euler's theorem (e.g., Pogorelov, 1974), average of curvatures of any two mutually perpendicular normal sections at a given point equals the mean curvature, *H*, of the surface, Table 2. In part, $H = 1/2(\mathrm{kmin} + \mathrm{kmax}) = 1/2(\mathrm{kh} + \mathrm{kv})$. The sense of this variable (of class C) can be explained using an example. If one takes a wire outline (not necessarily plane), when a soap film stretched on, it has minimal possible area for the given outline because surface energy tends to a minimum, in equilibrium conditions. This surface is called minimal (e.g., Guisti, 1984). It has been proven (Pogorelov, 1974) that in all points of the minimal surface, mean curvature is equal to zero. In this sense, small values of H mean that the surface is close to minimal — "equilibrium". The formula for H was known to Gauss (1827). In terms of Table 2, it follows from the formula of H that limit of this variable is $-1/2(r+t)$ as $p,q \rightarrow 0$; since the elevation Laplacian $\Delta Z = r + t$, this gives $\Delta Z \approx -2H$ for gently sloping terrains.

The unsphericity M of a surface is half of the difference between maximal and minimal curvatures (Table 2). Because kmax $\geq$ kmin, the unsphericity is non-negative. It equals zero on a sphere: its value shows the extent to which the geometrical form of surface is non-spherical. It follows from the expressions $H = 1/2(\text{kmin} + \text{kmax})$ and $K = \text{kmax} \cdot \text{kmin}$, that kmax,kmin $= H \pm (H^2 - K)^{1/2}$, and from here $M = 1/2(\text{kmin} - \text{kmax}) = (H^2 - K)^{1/2}$. Because formulae for H and K are known, we obtain the formula for M by transformation of the expression $(H^2 - K)^{1/2}$ to a sum of squares; the result is shown in Table 2.

Difference curvature E is equal to half the difference between vertical and horizontal curvatures. It shows which accumulation mechanism is more active, and is also important for classifying land surface forms in class A (Shary, 1995). It follows from the expressions $H = 1/2(\text{kv} + \text{kh})$ and $E = 1/2(\text{kv} - \text{kh})$, that $E = H - \text{kh}$; substituting known formulae for H and kh in the latter expression, one obtains formula for E presented in Table 2.

As mentioned above, there are three independent curvatures. It has been proven (Shary, 1995) that they can be chosen as the mean curvature H, the difference curvature E, and the unsphericity M. For such a choice, all 12 curvatures of the complete system of curvatures are expressed through these three curvatures by means of simple relations (Table 3).

Rotor rot is the curvature of flow lines, which are perpendicular to contour lines on a map. Rotor is positive when a flow line turns clockwise, and negative in an opposite case (Table 2). Rotor characterizes the twisting of flow lines, and does not belong to the complete system of curvatures. The formula for rot was first deduced in Shary (1991).

Vertical excess curvature (kve) is the difference between kv and the minimal curvature of normal section possible at the same point, that is kmin. So, kve describes to what extent kv is larger than kmin. Similarly, horizontal excess curvature (khe) describes to what extent kh is larger than kmin. Note that kve and khe are non-negative because curvature of any normal section at a given point is not less than kmin. These MVs were first introduced in Shary (1995); the formulae of Table 2 for them follow from Table 3.

5.4.1. Total ring curvature

Total ring curvature (KR) is defined as the product of kve and khe, Table 2, and is non-negative because kve,khe ≥ 0. It was proven (Shary, 1995, Theorem 6) that KR is equal to

Table 3
The relationships between the remaining curvatures of the complete system of curvatures and the three independent curvatures H, E, M

Curvatures of units 1/m		Total curvatures (of units 1/m^2)
kmax $= H + M$	kh $= H - E$	$K = H^2 - M^2$
kmin $= H - M$	kve $= M + E$	KA $= H^2 - E^2$
kv $= H + E$	khe $= M - E$	KR $= M^2 - E^2$

zero for any radially symmetrical form of surface with a vertical axis of symmetry. Using deduced above formulae $M=(H^2-K)^{1/2}$, $E=H-\mathrm{kh}$ and formula $\mathrm{KR}=M^2-E^2$ of Table 3, one obtains $\mathrm{KR}=2\mathrm{kh}\,H-\mathrm{kh}^2-K$; substituting here formulae for H, kh and K of Table 2 results in the formula for KR presented in Table 2. Comparing this with formula for rot, Table 2, one obtains $\mathrm{KR}=\mathrm{rot}^2G^2/(1+G^2)^2$. This result means that KR characterizes flow lines twisting without taking into account, turns a flow line clockwise or counterclockwise.

Total accumulation curvature (KA) is defined as the product of kv and kh (Table 2). It has been proven (Shary, 1995, Theorem 7) that KA is equal to zero for any surface forms that are described in polar coordinates as a function of polar angle, which does not depend on radius. Many such forms may be considered as constructed by passing rays from some point, and hence (because of straight slope profiles in such cases), the vertical curvature is equal to zero for them; KR, too, is zero. It is important that KA is positive in relative accumulation zones, where both the accumulation mechanisms act simultaneously, and in relative deflection zones (Shary, 1995), where both these mechanisms do not act. It is negative in remaining zones, where only one of these mechanisms acts. So, kv and kh permit construction maps of relative accumulation zones for substances that migrate on the land surface under gravity. Lanyon and Hall (1983) have used relative accumulation zones map for soil cover instability prediction.

Sometimes specialists subdivide the land surface into a random (noise) and systematical components (e.g., Clarke, 1988; Tomer and Anderson, 1995). For example, a weakly expressed spur of a hill may be essentially masked by numerous smaller hills, regarded as "noise". The sensitivity of local MVs to the random (noise) surface component is not equal, and increases in the following order: elevation, first derivatives, second derivatives of units 1/m, second derivatives of units 1/m^2, third derivatives (Shary and Stepanov, 1991). Application of the latter for land surface description seems to be not feasible because of their excessive sensitivity to "noise".

To obtain more expressive raster maps of MVs (or for comparisons with soil properties), their values can be logarithmed using the formula:

$$\mathrm{sign}(k)\ln(1+\mathrm{const}\,|k|), \tag{1}$$

where k is a morphometric variable, $\mathrm{sign}(k)=1$ for $k>0$, $\mathrm{sign}(k)=0$ for $k=0$, and $\mathrm{sign}(k)=-1$ for $k<0$. The value of const is empirical, and may be chosen equal to 10^6 for curvatures of units 1/m, and 10^{12} for total curvatures; for slope steepness, maximal catchment area and maximal dispersive area (see below) const = 9 may be chosen. Using the function sign(k) permits retention of the initial sign of k, after taking logarithm. Usage of 1 in the logarithm permits avoidance of infinity which is not acceptable for computer calculations because $\ln(0)=-\infty$.

5.5. Solar irradiation characteristics

Variables of classes A and C, described above, are designed for use in geometric land surface form analysis and for analysis of morphometric pre-requisits of dry and liquid surface runoff. To analyse the thermal regime of slopes, methods of classes A and B for the system "land surface + solar irradiation" are useful.

One such method consists in calculation of slope insolation F maps (Table 2). In a local consideration, F is an MV of class A; it represents solar radiation power expressed in percent (from 0 to 100%) of the maximally possible that is reached for a solar ray direction perpendicular to the land surface. Due to the local nature of F, hills may have shady side, but cannot produce shadows themselves (because shadow is a regional concept). Because F is a variable of class A, it has no sense without taking vector field of solar irradiation into account, and therefore depends upon the Sun's position, described by two angles: by an azimuth, and by an angle of Sun from the horizon. Because F is percent of flux power, that is, from energy per unit time per unit area (W/m^2), slope insolation characterizes the instantaneous Sun position. The angle of the Sun's position for a given latitude and time of year is known from astronomy, so the azimuth and angle from the horizon for the Sun can be determined by time of year and latitude. To obtain slope insolation not in percent, but in W/m^2, it is enough to multiply F/100 by Sun flux power at the land surface. For a sunny day and Sun position near zenith, this factor is equal to 2.25 kW/m^2 (Matveev, 1984). Disadvantages of F are: (a) it is not quite clear, for which time of day insolation best characterizes the diurnal average thermal regime of slopes, (b) F describes only direct (not diffused) solar irradiation.

An example of a slope insolation map is shown in Fig. 5b, where this map was used to "portray" land surface. For this purpose, the source of light was placed to the North (where the Sun is absent), because in the opposite case some subjective effects of poor visual perception may arise (ridges may be seen as valleys and vice versa; turn upside down this map to see these effects). In the GIS "ECO", values of all morphometric variables are available in each matrix element, and vector layers (hydrology, vegetation, buildings, etc.) can be displayed above a map image of most MVs.

More effective estimation of the thermal regime of slopes can be achieved by use of an MV that may be termed daily solar radiation dose. This determines not the power of the solar flux, but energy that the land surface receives daily, taking into account climate and state of atmosphere. This class B MV may be deduced by integration of the Sun's movement relative to the Earth for a day; formulae for it can be obtained by generalization of the astronomic theory of climate, known in meteorology (Matveev, 1984). The daily solar radiation dose (its units are J/m^2) may also use some meteorological data to account state of atmosphere and distinguish between direct and diffused solar radiation (see details in Matveev, 1984).

5.6. Regional morphometric variables

Maximal catchment area (MCA), class B, shows for a given matrix element (square on a map) the maximum area from which material moving downslope may be collected. In practice, MCA (units m^2) reflects both potential and realized hydrological channel network (without distinguishing them); during calculation, locations and depths of all B-depressions are also computed (Martz and de Jong, 1988; Costa-Cabral and Burges, 1994). An example of an MCA map is shown in Fig. 5e. Note that for calculation of the maximal dispersal area (Speight, 1974), which describes on which maximal area materials moving downslope may be diffused, there is no need to introduce two separate algorithms, as reported in Costa-Cabral and Burges (1994). It is enough for this purpose to invert the elevation matrix, and to use the same algorithm as for MCA on the inverted matrix (e.g., McKenzie and Ryan, 1999). Martz and de Jong (1988) have considered also a so-called minimal catchment area, for which all flow lines that reach floors of B-depressions disappear. This MV conceptually corresponds to the limit case of unfilled B-depressions (water evaporates at their bottoms); its essential disadvantage is the lack of contributing area integration along long channels and therefore excessive dependence of values on elevation minima.

As indicated in Introduction, MVs of classes A and B are chosen vector field action pre-requisits. Wind, snow melting, evapotranspiration, diffusion, inertia (e.g., for water streams), human activities are commonly known examples of other pre-requisits.

6. On methods of the class B

As indicated above, regional MVs principally need the map boundary to be taken into account for a terrain piece Ω, which is impossible in a planetary consideration (Maxwell, 1870) because the closed Earth surface has no boundaries. Speight (1968) has suggested an idea of determination of the specific catchment area, SCA, which is based on a consideration of a restricted terrain portion. Conceptually, SCA is contributing area per unit length of contour line. It was mentioned above that contour lines have no scale-free length, so one may use something like MCA/(grid mesh) to estimate SCA. The idea of catchment area received further development in some later works (Beven and Kirkby, 1979; Martz and de Jong, 1988; Freeman, 1991; Costa-Cabral and Burges, 1994). Martz and de Jong (1988) generalized Speight's concept of catchment area by taking B-depressions into account and re-discovering the idea of Maxwell (1870) that for each land surface point there exists a set of connected points, from which a continuous path with decreasing elevation can be found (the surface need not be smooth in this set definition). This set was termed a "dead zone" by Martz and de Jong (1988). Freeman (1991) and Quinn et al. (1991) suggested to take into account not only confluence, but also branching of water flows on land surface heterogeneities; this has allowed description of surface runoff more realistically and effectively reduced grid direction bias on MCA maps. Grid directions are explicitly seen on catchment area maps where flow branching is not taken into account (see Costa-Cabral and Burges, 1994). Beven and Kirkby (1979) have developed a simple model of basin hydrology trying to separate land surface and soil water storage effects; this has resulted in a so-called topographic index ln(SCA/tan(GA)) that obviously tends to infinity with GA $\rightarrow 0$, and therefore cannot be considered as a subject of the general geomorphometry.

6.1. Scale-free property of maximal catchment area

Essentially MCA is a result of an integro-differential transformation of a DEM: differentiation is necessary for flow direction determination, while integration is needed to take extended basin portions into account. Because of integral properties of this MV, one may expect that it may appear scale-free, at least in some terrain parts. Consider now this in more detail. The dependence of maximal value of MCA on grid mesh can be

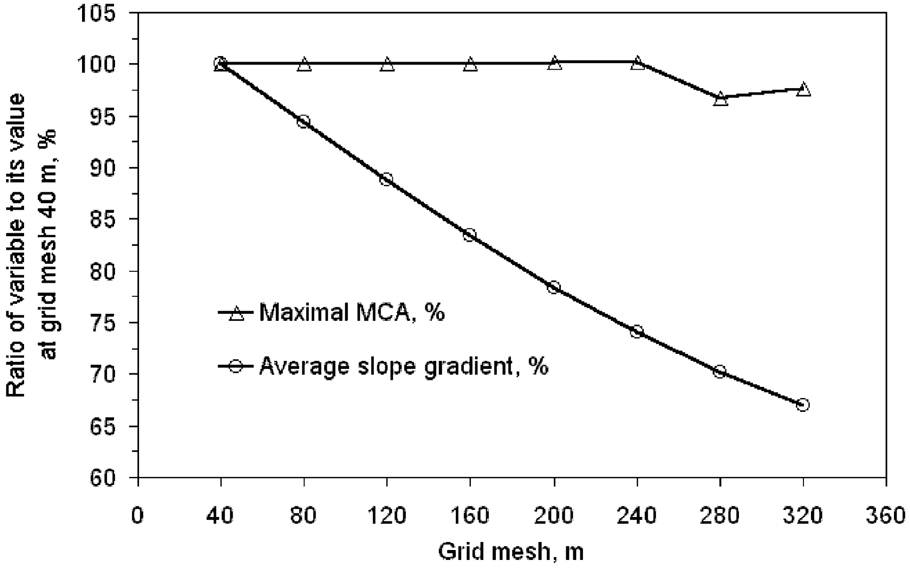


Fig. 7. The dependence of maximal value of MCA and average slope gradient on grid mesh w for mountainous river Alma basin, southern part of Crimea Peninsula, the Ukraine. The scale of initial topographic map is 1:200,000. For both the variables, their ratios (in %) to their initial value at grid mesh 40 m (64,575 km^2 and 0.08978, respectively) are shown. It is seen that max{MCA} practically does not depend on w, while the average slope gradient essentially decreases with w.

studied using a technique of matrix thinning; the results are shown in Fig. 7, where average slope gradient was used for a comparison (matrix size was 1033 by 1636 elements).

This result shows that max{MCA} for a given terrain does not depend on grid mesh w for sufficiently small w. max{MCA} is essentially area of largest terrain basin; hence, this morphometric characteristic is of integral type, not of integro-differential. This result will be valid for any point of main water course because appropriate map boundaries always can be chosen to make that point boundary. It is important in this consideration that MCA map of the main stream should be one matrix element width in points of interest because in opposite cases, max{MCA} may not represent basin area due to essential flow branching. Excluding such cases, we may resume that at least on main water streams MCA is a scale-free MV.

6.2. Hydrological aspects of some regional MVs

The description of surface runoff by a single morphometric variable (i.e., without physical modeling in space and time) means the restriction of landscape processes consideration by such situations, in which explicit dependence on time is absent. There are two natural situations of such a kind in surface water hydrology: (1) consequences of a single storm, and (2) steady state surface runoff characteristics that imply stationary conditions of rainfall.

In the first situation, after all water has passed outside terrain boundaries, the quantity of water that passed through each square on the terrain map (i.e., matrix element) does not depend upon time (under some additional suggestions). Consider a case of spatially uniform rainfall; to make this quantity independent from storm characteristics (such as intensity and duration), volume units should be replaced by an area, on which this water falls. This variable is just MCA; discrete forms here are B-depressions with horizontal free water surfaces that are considered as filled by water to maximally possible depths, that is, to the lowest outlet elevation (Fig. 1, point N). MCA calculation is usually accomplished by calculation of B-depression depth map, so GIS "ECO" can show for each pixel of an MCA map (Fig. 5e) also B-depression depth. Because MCA calculation uses essentially only flow line directions on a map, it describes only the first accumulation mechanism, in its regional expression.

In the second situation, an opportunity appears to describe both the accumulation mechanisms by a single MV, considering stationary runoff characteristics. Note that the topographic index ln(SCA/tan(GA)) does not describe new discrete forms. However, the latter are a fundamental concept of regional classes; in this case, the new discrete forms are dynamic depressions with non-horizontal free water surfaces. Rivers may be considered as examples of such depressions (for non-stationary conditions). Since river depths depend upon the rainfall intensity, a corresponding morphometric model must include dependent on this intensity parameter and describe dynamic depressions together with their depths. Such variable may be termed hydrodynamic area (HA, units m^2); more detailed description of this variable will include difference equations outside dynamic depressions and algorithm description inside them (Shary, in preparation). Although HA existence and features are not sufficiently described here, we have mentioned this idea because of its potential importance in soil science. Indeed, relatively slow slope profile changes are known as being responsible for numerous soil and parent material property changes (e.g., Ruhe, 1960, 1975; Pennock and de Jong, 1990; Coppinger et al., 1991; Moore et al., 1993; Fiez et al., 1995; Anjos et al., 1998; Bergstrom et al., 1998; Kleber et al., 1998), while regional MVs for their description (i.e., for second accumulation mechanism description) are not known at the time.

7. Geometrical form and memory in soils

For the climatic conditions of Israel, Sinai et al. (1981) have found a strong correlation ($r = 0.90$) between soil moisture and elevation Laplacian ΔZ that is approximately equal to $-2H$ for gently sloping terrains (as shown above); a similar result for soil moisture and H ($r = -0.89$) was obtained by Kuryakova et al. (1992) for humid climatic conditions of the Moscow region, Russia. The latter authors also showed that H correlates with soil moisture more strongly than kv or kh taken singly. Timlin et al. (1998) studied relationships between corn grain yield and ΔZ. They found that intra-annual differences in weather had the largest effect on grain yield at locations with $\Delta Z > 0$ (i.e., with $H < 0$), that is, predominantly in relative accumulation zones. So, these studies have revealed strong correlations between geometrical form attributes and some soil properties, such as soil moisture.

Such results may seem counter-intuitive: MVs of class C, which "ignore gravity", demonstrate stronger correlations with soil moisture than MVs of class A that describe surface runoff. Consider now a hypothetical situation that may elucidate some aspects of possible role of geometrical forms in soil science.

7.1. A hypothetical example

Suppose that some valley was inclined by land surface formation processes, as schematically shown in Fig. 8.

Now largest-intensity surface runoff pathway (thalweg) is changed according to the new orientation of this valley in gravitational field. Nevertheless, some "history-specific" soil properties, such as fine clay percent, may remain unchanged in the old thalweg location, because some time interval is needed for soil to "forget" its pre-history stored in clay percent. In other words, comparison of geometrical form attributes (class C) and those of surface runoff (class A) is able to provide some information on this specific mechanism of memory in soils; note that class C MVs are invariant to surface rotations. It follows from the equality $H = 1/2(\text{kmin} + \text{kmax})$ obtained above and $\text{kmax} = 0$, $\text{kmin} < 0$ for this situation that $H = \text{kmin}/2$; since $|\text{kmin}|$ is now largest in the old thalweg location (in contrast to MCA), kmin or H will probably stronger correlate with such history-specific soil properties. From the other side, the gravitational field controls only surface runoff, not directly soil moisture, because the latter also depends on other soil properties, such as soil specific surface area (e.g., Petersen et al., 1996) that is a function of clay percent. As a result, soil moisture may more strongly correlate with kmin or H than with surface runoff attributes, such as kv, kh, or MCA.

7.2. General notes

The situation described is only one concrete example (for this hypothetical situation), when field-invariant MVs may appear more effective in soil property prediction than field-specific MVs. Generally speaking, in processes of land surface formation, geometrical

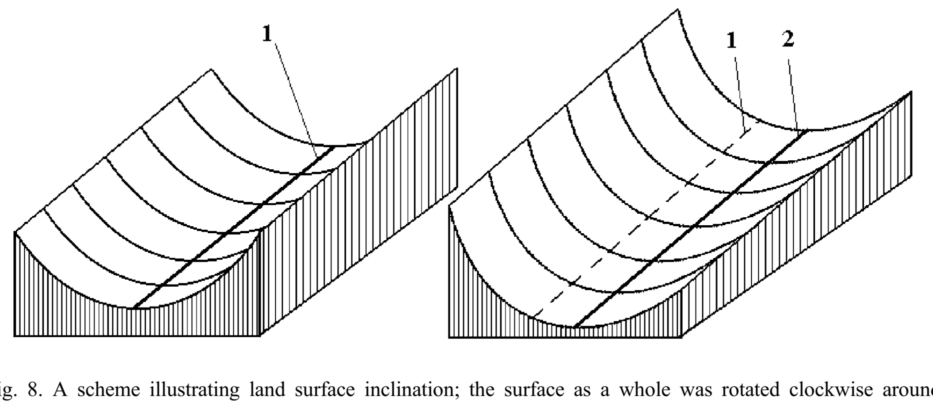


Fig. 8. A scheme illustrating land surface inclination; the surface as a whole was rotated clockwise around thalweg. Left — old surface orientation, right — present one. 1 — old thalweg location, 2 — new one.

forms control flows and flows modify forms; land surface formation is accomplished by soil formation and field-invariant MVs (i.e., geometrical form attributes) seem to be useful in soil science due to their ability (in conjunction with gravity-specific MVs) to extract information on specific mechanisms of memory in soil, which reflect effects of temporal shifts between land surface formation and soil formation processes. The extent of their practical value depends on how important is this kind of memory in soils; the latter seems to be a subject of corresponding experimental studies, not only of theoretical considerations. In this context, geometrical form analysis provides new tools to obtain some facts that refer to this kind of memory in soils.

Using only gravity-specific MVs in terrain studies, soil scientists implicitly consider land surface formation (accomplished by soil formation) as occurring under only gravity-induced surface runoff influence; wind, tectonics and other pre-requisits of land surface formation are essentially ignored. Gile (1999) has described a terrain, where wind impact is considered as one of major pre-requisits of land surface formation; real contributions of various pre-requisits often stay unknown. It seems to be too difficult task to consider all of them separately; consequently, the following question arise: which MVs are able to take all of them into account? We believe that field-invariant MVs are, because geometrical forms are result of all pre-requisits of land surface formation (even of those that result in up-slope material movement, such as wind erosion). This is an invariant, complementary to field-specific representation of land surface analyses; in any case, morphometric analysis with no use of field-invariant variables is not complete.

8. Discussion

The current state of general geomorphometry permits analysis at a local level of geometrical land form, surface runoff, and the thermal regime of slopes. Although the theory of local approaches in its major part can be considered as completed by proof of the completeness of the system of 12 curvatures (Shary, 1995), its possibilities are far from being sufficiently studied in soil science. It is hoped that use of this expanded system of morphometric variables (MVs) is able to provide larger information (and show stronger correlations) for natural relationships studying than frequently used approaches that are based on mentioned above not substantiated notion, which the number of independent curvatures is two, not three.

We believe that there exists a generalization to regional approaches for most local MVs of general geomorphometry. For example, MCA generalizes in some sense to the class B the local description of the first accumulation mechanism by the horizontal or plan curvature. From this point of view, the theory of regional MVs is not sufficiently developed yet; in addition, although the role of known regional MVs is sufficiently well recognized even at present (e.g., Moore et al., 1993; McKenzie and Ryan, 1999), their fuller introduction and systematic use may provide more powerful new tools for soil science. The conclusion of this paper that some regional concepts (such as B-depressions, B-hills, MCA on thalweg areas) have precise (scale-free) values is also an argument for this belief. Scale-free property of regional MVs is of great importance in soil science because it makes comparable results obtained at different scales. The results of this study show that scale-free property of new MVs is expected in regional classes B or D; nevertheless, this property does not follows from that an MV belongs to one of these classes. For example, scale-free property of MCA on thalwegs is lost for ln(MCA/G) because slope gradient G is not scale-free.

As mentioned above, relatively slow profile changes are of great importance in soil science. Consequently, the absence of regional MVs for the second accumulation mechanism description should be considered as an essential disadvantage of present-day general geomorphometry.

8.1. On ambiguity in land form definitions

Land form classifications have not been considered in this paper. Nevertheless, some conclusions on land form types can be made from the above consideration of classes of MVs.

Aandahl (1948) has defined convergent areas as concave spurs and divergent areas as convex spurs; he termed areas of relative deceleration as concave terraces, and areas of relative acceleration (where vertical curvature is positive) as convex terraces. Note that these definitions refer to class A, that is, these are concave A-spurs, convex A-terraces, and so on. Many intuitive definitions in geomorphology have up to four quantitatively definable interpretations, not more than one in each class. So, a depression can be defined by the main normal sections as a concave–concave part of a surface (Gauss, 1827), or as a relative accumulation zone (Shary, 1995), or as a B-depression (Martz and de Jong, 1988), or as a D-depression (Fig. 1a). These four definitions are essentially definitions of the four non-overlapping classes of Table 1; they are mutually complementary, and there is a definite relationship between them. In this concrete example, a B-depression is a part of D-depression that is able to hold water (Fig. 1), and a C-depression is a part of an A-depression, as have been proven in Shary (1995). Elevation matrix inversion allows four definitions to be given for a hill also. Such ambiguity of definitions is an intrinsic feature of geomorphometric concepts, and sometimes results in different understanding of the same words or terms by different authors, but this ambiguity is restricted to the four non-overlapping classes of concepts, at least, until global (planetary) concepts are not considered.

8.2. On new morphometric variables introduction

This paper is essentially devoted to the topics related to where (in which class) and how (by curvature concept generalization) new scale-free MVs might be introduced in general geomorphometry, formal description of curvature was published earlier (Shary, 1995). Consider now some tasks that are to be solved for slow profile changes description.

Essentially, the general task is that of Aandahl's A-terrace concept (or vertical curvature concept) generalization to the class B. Terraces are perpendicular in general to slope direction and they may have more or less frequent interrupts that may be interpreted as a "noisy" component of the land surface. A fixed-size filter of class A used to delineate A-terraces may not sum non-interrupting terrace fragments because they are defined by terrain specifics, not by a fixed filter. A "searching" filter of class B should do this. Another task is to search also B-terrace widths; with fixed filters, this is done only by maps observer, visually. Discrete forms are subject of a special consideration; mentioned above dynamic depressions appear as a result of morphometric modeling, being not directly geometrical concepts: they depend on stationary rainfall intensity.

Too small number of currently introduced scale-free MVs has resulted in that researchers have a stable belief that for each geological structure or land form its own scale is to be applied (Klemeš, 1983; Clarke, 1988; Phillips, 1988), or that terrain consideration at several different scales cannot be replaced by its consideration at a single sufficiently large scale. Although only three of curvatures are independent, an opportunity that curvature concepts generalization to classes B and D will result in a larger number of independent MVs of a new system of regional MVs cannot be disclaimed a priory.

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Appendix A. The modified Evans-Young method

Evans–Young method consists of the following. The second-order polynomial $z = rx^2/2 + sxy + ty^2/2 + px + qy + z_0$ is fitted by the least squares method to the subgrid 3×3, with grid mesh w and elevations in nodes $z_1, \ldots, z_9$. The locations and numbering of nodes are shown in Fig. A1.

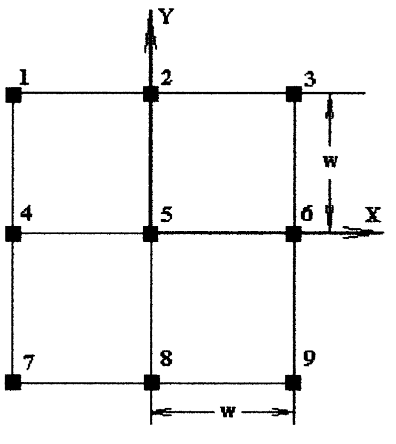

Fig. A1. Locations and numbering of nodes in subgrid 3 × 3.

This gives the following formulae for coefficients p, q, r, s, t of the polynomial:

$$p = (z_3 + z_6 + z_9 - z_1 - z_4 - z_7)/6w,$$
$$q = (z_1 + z_2 + z_3 - z_7 - z_8 - z_9)/6w,$$
$$r = [z_1 + z_3 + z_4 + z_6 + z_7 + z_9 - 2(z_2 + z_5 + z_8)]/3w^2,$$
$$s = (-z_1 + z_3 - z_7 + z_9)/4w^2,$$
$$t = [z_1 + z_2 + z_3 + z_7 + z_8 + z_9 - 2(z_4 + z_5 + z_6)]/3w^2;$$

see details on deduction of these formulae in Young (1978) and Pennock et al. (1987).

The coefficients p, q, r, s, t are partial derivatives of the polynomial at the central point of the subgrid that has coordinates $x = y = 0$. Namely, $p = \partial z/\partial x$; $q = \partial z/\partial y$; $r = \partial^2 z/\partial x^2$; $s = \partial^2 z/\partial x \partial y$; $t = \partial^2 z/\partial y^2$. Now, using these values of partial derivatives, one may calculate for the central (fifth) point of the subgrid 3×3 any local morphometric variables that are expressed by first and second derivatives from elevations of the surface given by the equation $z = z(x,y)$. Then, the subgrid 3×3 is moved by its center to another element of the elevation matrix, and the procedure is repeated. Calculations are carried out for all but boundary matrix elements.

The modification of Evans–Young method consists in that before calculations by this method, a parametric isotropic smoothing of the initial elevation matrix is performed. This smoothing is realized by replacing the elevation z_5 in the central point (fifth, Fig. A1) by a new value z_5', which is linearly expressed through 3×3 subgrid points elevations:

$$z_5' = a_1 z_1 + a_2 z_2 + a_3 z_3 + a_4 z_4 + a_5 z_5 + a_6 z_6 + a_7 z_7 + a_8 z_8 + a_9 z_9. \tag{1A}$$

There are nine unknown weights, $a_1, \ldots, a_9$, for the filter (Eq. (1A)). The first condition for their determination consists in that (A) *a plane piece of surface is transformed into a plane one*. An equation of a plane is $z = \alpha x + \beta y + z_5$; substituting into this formula coordinate values x, y of subgrid nodes (Fig. A1), one obtains:

$$z_1' = w(-\alpha + \beta) + z_5, \qquad z_2' = \beta w + z_5, \qquad z_3' = w(\alpha + \beta) + z_5,$$
$$z_4' = -\alpha w + z_5, \qquad z_5' = z_5, \qquad z_6' = \alpha w + z_5,$$
$$z_7' = w(-\alpha - \beta) + z_5, \qquad z_8' = -\beta w + z_5, \qquad z_9' = w(\alpha - \beta) + z_5;$$

it follows from this for an arbitrary plane

$$\begin{aligned} z_5' &= \alpha w(-a_1 + a_3 - a_4 + a_6 - a_7 + a_9) \\ &\quad + \beta w(a_1 + a_2 + a_3 - a_7 - a_8 - a_9) + z_5 \Sigma a_i \end{aligned}$$

Left part of this equation must be equal to z_5 because the filter does not change elevations for the plane:

$$(1 - \Sigma a_i)z_5 = \alpha\, w(-a_1 + a_3 - a_4 + a_6 - a_7 + a_9) \\ + \beta\, w(a_1 + a_2 + a_3 - a_7 - a_8 - a_9). \quad (2A)$$

Because $a_1,\ldots,a_9$ are numbers and the last equality should be valid for any α, β, z_5, all the three parentheses in Eq. (2A) must be equal to zero. From here:

$$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 = 1, \quad (3A)$$

$$a_3 + a_6 + a_9 - a_1 - a_4 - a_7 = 0, \quad a_1 + a_2 + a_3 - a_7 - a_8 - a_9 = 0. \quad (4A)$$

The next condition is that (B) *the filter is isotropic*. This means that all weights are dependent only from the distance between node and central point. One finds from here the equalities $a_1 = a_3 = a_7 = a_9$, $a_2 = a_4 = a_6 = a_8$, from which Eq. (4A) follow automatically; the Eq. (3A) takes now the form:

$$a_5 = 1 - 4(a_1 + a_2). \quad (5A)$$

The filter (Eq. (1A)) is now $z_5' = a_1(z_1 + z_3 + z_7 + z_9) + a_2(z_2 + z_4 + z_6 + z_8) + a_5 z_5$; substituting here a_5 from Eq. (5A), one finds:

$$z_5' = a_1(z_1 + z_3 + z_7 + z_9) + a_2(z_2 + z_4 + z_6 + z_8) + [1 - 4(a_1 + a_2)]z_5. \quad (6A)$$

So, the conditions (A) and (B) leave unknown the two weights of the filter (Eq. (1A)): a_1 and a_2. One from them can be defined by the condition of (C) *linearity of weights decreasing with a distance from the central point*. Let us rewrite Eq. (6A) for this in the form:

$$z_5' = k(z_1 + z_3 + z_7 + z_9)/9 + s(z_2 + z_4 + z_6 + z_8)/9 + [1 - 4(k + s)]z_5,$$

where the smoothing parameter $s \in [0,1]$, and the condition (C) gives for k the expression:

$$k = 1 - 2^{1/2}(1 - s) \text{ for } s \in [1 - 2^{-1/2}, 1] \quad \text{and}$$

$$k = 0 \text{ for } s \in [0, 1 - 2^{-1/2}].$$

The remaining free parameter s determines extent of surface smoothing by the filter. An absence of smoothing ($z_5' = z_5$) corresponds to the value $s = 0$, non-weighted average $z_5' = \Sigma z_i/9$ of the grid points corresponds to the value $s = 1$. At $s < 1 - 2^{-1/2} \cong 0.293$, the expression for z_5' is simplified because of $k = 0$:

$$z_5' = s(z_2 + z_4 + z_6 + z_8)/9 + (1 - 4s/9)z_5.$$

It was empirically stated that $s=1/5$ ("weak" smoothing) gives good results for maps of curvatures practically for any terrain. The last formula takes for this value the form:

$$z_5' = (z_2 + z_4 + 41z_5 + z_6 + z_8)/45.$$

Smoothing of 9 points of the subgrid 3×3 by this filter with a subsequent usage of Evans–Young method to this smoothed subgrid results in the modified Evans–Young method.

References

Aandahl, A.R., 1948. The characterization of slope positions and their influence on the total nitrogen content of a few virgin soils of Western Iowa. Soil Science Society of America Proceedings 13, 449–454.

Anjos, L.H., Fernandes, M.R., Pereira, M.G., Franzmeier, D.P., 1998. Landscape and pedogenesis of an Oxisol–Inceptisol–Ultisol sequence in southeastern Brazil. Soil Science Society of America Journal 62, 1651–1658.

Bell, J.C., Cunningham, R.L., Havens, M.W., 1992. Calibration and validation of a soil-landscape model for predicting soil drainage class. Soil Science Society of America Journal 56, 1860–1866.

Bell, J.C., Cunningham, R.L., Havens, M.W., 1994. Soil drainage class probability mapping using a soil-landscape model. Soil Science Society of America Journal 58, 464–470.

Bergstrom, D.W., Monreal, C.M., King, D.J., 1998. Sensitivity of soil enzyme activities to conservation processes. Soil Science Society of America Journal 62, 1286–1295.

Beven, K.J., Kirkby, M.J., 1979. A physically based, variable contributing area model of basin hydrology. Hydrological Sciences Bulletin 24, 43–69.

Carter, B.J., Ciolkosz, E.J., 1991. Slope gradient and aspect effects on soils developed from sandstone in Pennsylvania. Geoderma 49, 199–213.

Clarke, K.C., 1988. Scale-based simulation of topographic relief. The American Cartographer 15, 173–181.

Coppinger, K.D., Reiners, W.A., Burke, I.C., Olson, R.K., 1991. Net erosion on a Sagebrush steppe landscape as determined by cesium-137 distribution. Soil Science Society of America Journal 55, 254–258.

Costa-Cabral, M.C., Burges, S.J., 1994. Digital elevation model networks (DEMON): a model of flow over hillslopes for computation of contributing and dispersal areas. Water Resources Research 30, 1681–1692.

Evans, I.S., 1972. General geomorphometry, derivatives of altitude, and descriptive statistics. In: Chorley, R.J. (Ed.), Spatial Analysis in Geomorphology. Methuen & Co., London, pp. 17–90, Chap. 2.

Evans, I.S., 1975. The effect of resolution on gradients calculated from an altitude matrix. Report 3 on Grant DA-ERO-591-73-G0040, 'Statistical characterization of altitude matrices by computer' (Appendix: Stationarity). Department of Geography, University of Durham, England, 24 pp.

Evans, I.S., 1979. An integrated system of terrain analysis and slope mapping. Final Report (Report 6) on Grant DA-ERO-591-73-G0040, 'Statistical characterization of altitude matrices by computer'. Department of Geography, University of Durham, England, 192 pp.

Evans, I.S., 1980. An integrated system of terrain analysis and slope mapping. Zeitschrift fuer Geomorphologie N.F., Supplementband 36, 274–295.

Evans, I.S., 1990. General geomorphometry. In: Goudie, A. (Ed.), Geomorphological Techniques. 2nd edn. Allen & Unwin, London, pp. 44–56, Chap. 2.3.

Evans, I.S., Cox, N.J. et al., 1999. Relations between land surface properties: altitude, slope and curvature. In: Hergarten, S., Neugebauer, H.J. (Eds.), Process Modelling and Landform Evolution. Springer, Berlin, pp. 13–45.

Fiez, T.E., Pan, W.L., Miller, B.C., 1995. Nitrogen use efficiency of winter wheat among landscape positions. Soil Science Society of America Journal 59, 1666–1671.

Finn, R., 1986. Equilibrium Capillary Surfaces. Springer-Verlag, New York, 312 pp.

Freeman, T.G., 1991. Calculating catchment area with divergent flow based on a regular grid. Computers & Geosciences 17, 413–422.

Gauss, C.F., 1827. Disquisitiones generales circa area superficies curvas. Gott. Gel. Anz. 177, S1761–S1768 (in Latin).

Gile, L.H., 1999. Eolian and associated pedogenic features of the Jornada Basin floor, southern New Mexico. Soil Science Society of America Journal 63, 151–163.

Guisti, E., 1984. Minimal Surfaces and Functions of Bounded Variation. Birkhäuser, Boston, 240 pp.

Huang, C., 1995. Empirical analysis of slope and runoff for sediment delivery from interrill areas. Soil Science Society of America Journal 59, 982–990.

King, D., Bourennane, H., Isampert, M., Macaire, J.J., 1999. Relationship of the presence of a non-calcareous clay-loam horizon to DEM attributes in a gently sloping area. Geoderma 89, 95–111.

Kleber, A., Mailänder, R., Zech, W., 1998. Stratigraphic approach to alteration in mineral soils: the heavy metal example. Soil Science Society of America Journal 62, 1647–1650.

Klemeš, V., 1983. Conceptualization and scale in hydrology. Journal of Hydrology 65, 1–23.

Klich, I., Wilding, L.P., Pfordresher, A.A., 1990. Close-interval spatial variability of Udertic Paleustalfs in east cantral Texas. Soil Science Society of America Journal 54, 489–494.

Koenderink, J.J., van Doorn, A.J., 1994. Two-plus-one-dimensional differential geometry. Pattern Recognition Letters 15, 439–443.

Krcho, J., 1973. Morphometric analysis of relief on the basis of geometric aspect of field theory. Acta Geographica Universitatis Comenianae, Geographico-Physica 1, 7–233.

Krcho, J., 1983. Teoretická concepcia a interdisciplinarne aplikacie komplexného digitalneho modelu reliéfu pri modelovanï dvojdimenzionalnych poli. Geograficky Casopis 35, 265–291 (in Slovak).

Kuryakova, G.A., Florinsky, I.V., Shary, P.A., 1992. On the correlation between soil moisture and certain topographic variables. In: Andronikov, V.L. (Ed.), The Modern Problems of Soil Geography and Mapping, Proceedings of National Conference, September 24–26, 1991. Dokuchaev Soil Institute, Moscow, pp. 70–71 (in Russian).

Lanyon, L.T., Hall, G.F., 1983. Land-surface morphology: 2. Predicting potential landscape instability in eastern Ohio. Soil Science 136, 382–386.

Mandelbrot, B., 1967. How long is the coast of Britain? Statistical self-similarity and fractional dimension. Science 156, 636–638.

Martz, L.W., de Jong, E., 1988. CATCH: a Fortran program for measuring catchment area from digital elevation models. Computers & Geosciences 14, 627–640.

Martz, L.W., de Jong, E., 1990. Natural radionuclides in the soils of a small agricultural basin in the Canadian Prairies and their association with topography, soil properties and erosion. Catena 17, 85–96.

Matheron, G., 1965. Les variables regionalisées et leur estimation. Masson, Paris, 305 pp.

Matheron, G., 1975. Random Sets and Integral Geometry. Wiley, New York, 261 pp.

Matveev, L.T., 1984. The Course of General Meteorology. Physics of Atmosphere. 2nd edn. Gidrometeoizdat, Leningrad, 752 pp. (in Russian).

Maxwell, J.C., 1870. On hills and dales. Philosophical Magazine, Series 4 40 (269), 421–427.

McKenzie, N.J., Ryan, P.J., 1999. Spatial prediction of soil properties using environmental correlation. Geoderma 89, 67–94.

Moore, I.D., O'Loughlin, E.M., Burch, G.J., 1988. A contour-based topographic model for hydrological and ecological applications. Earth Surface Processes and Landforms 13, 305–320.

Moore, I.D., Gessler, P.E., Nielsen, G.A., Peterson, G.A., 1993. Soil attribute prediction using terrain analysis. Soil Science Society of America Journal 57, 443–452.

Odeh, I.O.A., Chittleborough, D.J., McBratney, A.B., 1991. Elucidation of soil-landform interrelationships by canonical ordination analysis. Geoderma 49, 1–32.

Odeh, I.O.A., McBratney, A.B., Chittleborough, D.J., 1992a. Soil pattern recognition with fuzzy-*c*-means: application to classification and soil-landform interrelationships. Soil Science Society of America Journal 56, 505–516.

Odeh, I.O.A., McBratney, A.B., Chittleborough, D.J., 1992b. Fuzzy-*c*-means and kriging for mapping soils as a continuous system. Soil Science Society of America Journal 56, 1848–1854.

Onorati, G., Poscolieri, M., Ventura, R., Chiarini, V., Crucilla, U., 1992. The Digital Elevation Model of Italy for geomophology and structural geology. Catena 19, 147–178.

Pennock, D.J., de Jong, E., 1990. Regional and catenary variations in properties of Borolls of southern Saskatchewan, Canada. Soil Science Society of America Journal 54, 1697–1701.

Pennock, D.J., Zebarth, B.J., de Jong, E., 1987. Landform classification and soil distribution in hummocky terrain, Saskatchewan, Canada. Geoderma 40, 297–315.

Petersen, L.W., Moldrup, P., Jacobsen, O.H., Rolston, D.E., 1996. Relations between specific surface area and soil physical and chemical properties. Soil Science 161, 9–21.

Phillips, J.D., 1988. The role of spatial scale in geomorphic systems. Geographical Analysis 20, 308–317.

Pogorelov, A.V., 1974. The Differential Geometry. Nauka, Moscow, 176 pp. (in Russian).

Quinn, P., Beven, K., Chevalier, P., Planchon, O., 1991. The prediction of hillslope flow paths for distributed hydrological modelling using digital terrain models. Hydrological Processes 5, 59–79.

Ruhe, R.V., 1960. Elements of the soil landscape. In: Transactions of 7-th International Congress of Soil Science, International Society of Soil Science, Madison, WI, U.S.A., pp. 165–170.

Ruhe, R.V., 1975. Geomorphology. Geomorphic Processes and Surficial Geology. Houghton Miffin, Boston, 246 pp.

Shary, P.A., 1991. Topographic method of second derivatives. In: Stepanov, I.N. (Ed.), The Geometry of Earth Surface Structures. Poushchino Scientific Center, Poushchino, pp. 28–58 (in Russian).

Shary, P.A., 1995. Land surface in gravity points classification by a complete system of curvatures. Mathematical Geology 27, 373–390.

Shary, P.A., Stepanov, I.N., 1991. On the second derivative method in geology. Doklady Akademii Nauk SSSR 319, 456–460 (in Russian).

Simmons, F.W., Cassel, D.K., Daniels, R.B., 1989. Landscape and soil property effects on corn grain yield response to tillage. Soil Science Society of America Journal 53, 534–539.

Sinai, G., Zaslavsky, D., Golany, P., 1981. The effect of soil surface curvature on moisture and yield-Beer Sheba observations. Soil Science 132, 367–375.

Sinowski, W., Auerswald, K., 1999. Using relief parameters in a discriminant analysis to stratify geological areas with different spatial variability of soil properties. Geoderma 89, 113–128.

Speight, J.G., 1968. Parametric description of land form. In: Stewart, G.A. (Ed.), Land Evaluation. Macmillan, London, pp. 239–250.

Speight, J.G., 1974. A parametric approach to landform regions. Progress in GeomorphologyInstitute of British Geographers Special Publication, vol. 7. Alden & Mowbray at the Alden Press, Oxford, pp. 213–230.

Stepanov, I.N., 1996. Introduction of soil geological forms of flow structures to cartography. Eurasian Soil Science 28, 17–33.

Stepanov, I.N., Florinsky, I.V., Shary, P.A., 1991. On the conceptual scheme of landscape studies. In: Stepanov, I.N. (Ed.), The Geometry of Earth Surface Structures. Poushchino Scientific Center, Poushchino, pp. 9–15 (in Russian).

Strahler, A.N., 1952. Hypsometric (area-altitude) analysis of erosional topography. Bulletin of the Geological Society of America 63, 1117–1141.

Thomas, A.L., King, D., Dambrine, E., Couturier, A., Roque, J., 1999. Predicting soil classes with parameters derived from relief and geologic materials in a sandstone region of the Vosges mountains (Northeastern France). Geoderma 90, 291–305.

Timlin, D.J., Pachepsky, Ya., Snyder, V.A., Bryant, R.B., 1998. Spatial and temporal variability of corn grain yield on a hillslope. Soil Science Society of America Journal 62, 764–773.

Tomer, M.D., Anderson, J.L., 1995. Variation of soil water storage across a sand plain hillslope. Soil Science Society of America Journal 59, 1091–1100.

Troeh, F.R., 1964. Landform parameters correlated to soil drainage. Soil Science Society of America Proceedings 28, 808–812.

Webster, R., 1977. Quantitative and numerical methods in soil classification and survey. Oxford: Oxford University Press. 269 p.

Young, M., 1978. Terrain analysis: program documentation. Report 5 on Grant DA-ERO-591-73-G0040, 'Statistical characterization of altitude matrices by computer'. Department of Geography, University of Durham, England, 27 pp.