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THE EFFECT OF TRANSPORTATION SUBSIDIES ON URBAN SPRAWL*

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ABSTRACT. This paper provides theoretical and empirical analyses of the effect of transportation subsidies on urban sprawl in a two-mode urban spatial model. Comparative static analysis shows, among other things, that the urban area contracts with a public transit subsidy but expands with an auto subsidy. The paper provides the first empirical test of these hypotheses and finds that the spatial size of the urbanized area shrinks with an increase in transit subsidies but increases at a decreasing rate with auto subsidies.

1. INTRODUCTION

U.S. highway expenditures in 2000 were $127.5 billion, of which $81.0 billion was covered by highway user-fees. This resulted in a $46.5-billion subsidy to highway users (Federal Highway Administration, 2005). That motorists were subsidized at 36.5 percent of total highway expenditures in the year 2000 is not an anomaly. According to Voith (1989), during the period 1956 to 1986, motorists were subsidized at 32 percent of the U.S. highway system's capital and maintenance costs.

Using data on 518 public transit agencies for 2000 (Federal Transit Administration, 2005), we find fares amounting to $8.1 billion accounted for only 36 percent of operating costs and only 26 percent of operating and capital costs. In addition, we find that 82 percent of transit agencies cover 30 percent or less of their operating expenses from fare revenues and that only 4 percent of transit agencies report fare revenues in excess of 50 percent of operating costs.

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If users do not pay the full cost of their travel, they have an incentive to travel greater distances and make more frequent trips. Transportation subsidies, therefore, could be a source of urban sprawl.[1] Urban sprawl is a topic that has generated much debate in recent years and has become an important policy issue in the United States. In 1998, more than 150 ballot measures were introduced to restrict urban sprawl in one way or another, and more than 85 percent of those measures were passed (Staley, 1999). Many state and local governments now argue for using transportation regulations to curb urban sprawl. These regulations include increasing public transit subsidies and reducing highway expenditures, which indirectly reduces highway subsidies. For example, the Oregon Department of Transportation increased funding for public transit service, which was supported by 78 percent of Oregonians based on a statewide survey (Oregon Department of Transportation, 2007). Starting in 2007, the State of Illinois provides additional funding for transit agencies and antisprawl subsidies to companies that build close to affordable housing units or public transit routes (Illinois Department of Transportation, 2007; The New Standard, 2007).

Brueckner (2005) is the only person of whom we are aware to deal with transportation subsidies as a potential source of urban sprawl. We provide an extension of Brueckner's single-mode model by incorporating key institutional features that add to the model's realism. Brueckner also provides an analysis of system choice, an issue with which we do not deal in this paper. Brueckner assumes that the urban area has only one type of transportation system, while we develop a two-mode model to reflect the fact that in the majority of even the smallest urban areas, public transit is available for urban residents. Of the 201 urban areas on which we have data, 111 have transit agencies reporting to the FTA. To capture the fact that subsidy-induced deficits in the operation of the transportation system have to be covered by tax revenues, Brueckner assumes that households pay a head tax in his balanced budget equation. Our balanced budget equation includes an "income" tax, intended to capture all taxes paid by urban households to urban governments, and intergovernmental grants, which are a common source of funds to urban-area governments. In our model, we find an inverse relation between transit subsidies and sprawl and a direct relation between auto subsidies and sprawl. We also provide empirical evidence that the spatial size of the urbanized area contracts with an increase in transit subsidies and expands (but at a decreasing rate) with an increase in auto subsidies.

2. TRANSPORT SUBSIDIZATION

As shown in Section 1, U.S. highway and transit users are subsidized generously. It is necessary, however, to study these subsidies more closely because

[1] Mills (1999) and Brueckner (2001) define urban sprawl normatively as *excessive* decentralization of urban population. We use the term here to mean simply decentralization of urban population.

TABLE 1: U.S. Highway Revenue Sources and Expenditures, 2000

Category of Revenue and Expenditure	Expenditure and Revenue, by Government (billions) Federal	State	Local	All	Percent of Total
User charges					
Motor-fuel taxes	$25.1	$28.7	$1.0	$54.8	42.5
Motor-vehicle taxes and fees	4.6	15.5	0.7	20.8	16.2
Tolls	0.0	4.7	0.7	5.4	4.2
Subtotal	$29.7	$49.0	$2.3	$81.0	62.9
Other revenue sources					
Property taxes and assessments	$0.0	$0.0	$6.4	$6.4	4.9
General fund appropriations	1.2	4.1	11.9	17.2	13.4
Other taxes and fees	0.1	2.4	2.8	5.4	4.2
Investment income & other receipts	0.0	2.7	4.8	7.5	5.8
Bond issue proceeds	0.0	8.2	3.1	11.2	8.7
Subtotal	$1.4	$17.5	$28.9	$47.7	37.1
Total revenues	$31.1	$66.4	$31.3	$128.1	100.0
Funds drawn from (or placed in) reserve	($3.3)	$0.6	$1.5	($1.3)	−1.0
Total expenditures	$27.7	$67.0	$32.7	$127.5	99.0

Source: Federal Highway Administration, 2005 (*Highway Statistics*, Table HF-10).

the way in which urban area transportation is subsidized affects both the theory and empirics of this paper.

Table 1 provides data on U.S. highway revenues and expenditures. The federal government spent $27.7 billion on highways in 2000, while user charges were $29.7 billion. These figures reveal that the federal government provides no subsidies to highway users in aggregate. State governments spent $67.0 billion on highways but collected only $49.0 billion from users, so state governments subsidized highway users by $18.0 billion. Total highway expenditures by local governments were $32.7 billion, while highway user charges were only $2.3 billion, leaving a deficit of $30.4 billion.

Table 2 reveals that intergovernmental transfers play a very important role in highway finance. Although the federal government funded $27.7 billion, or 21.7 percent, of total highway expenditures in 2000, 98.2 percent of the federal government's contribution to highways consisted of grants to state and local governments. Total direct federal expenditures amounted to only $2.3 billion, or 1.8 percent of total expenditures. State governments combined $24.4 billion of federal funds with $52.1 billion of state funds and $1.3 billion of local funds to make highway expenditures of $77.9 billion, or 61.1 percent of total expenditures. Local governments combined $1 billion of federal funds with $14.9 billion of state funds and $31.4 billion of local funds to make expenditures of $47.3 billion, or 37.1 percent of total expenditures.

Table 3 provides similar data on public transit agencies. Total operating expenditures for public transit in 2000 were $22.6 billion, of which only 35.9

TABLE 2: U.S. Highway Expenditures by Agency and Type, 2000

	Expenditure, by Government (billions)				
Expenditure Type and Funding Source	Federal	State	Local	All	Percent
Capital expenditures					
Funded by federal government	$0.3	$24.4	$1.0	$25.8	20.2
Funded by state or local government	0.0	23.2	15.7	38.9	30.5
Subtotal	$0.3	$47.6	$16.7	$64.7	50.7
Operating expenditures					
Maintenance	$0.2	$9.1	$14.9	$24.2	19.0
Highway and traffic services	0.0	3.8	2.9	6.8	5.3
Administration	1.8	5.5	3.0	10.3	8.1
Highway patrol and safety	0.0	5.7	5.0	10.7	8.4
Interest on debt	0.0	3.0	2.0	5.1	4.0
Bond retirement	0.0	3.1	2.7	5.7	4.5
Subtotal	$2.1	$30.2	$30.5	$62.8	49.3
Total expenditures	$2.3	$77.8	$47.2	$127.5	100.0
Funding source					
Federal government	$2.3	$24.4	$1.0	$27.7	21.7
State governments	0.0	52.1	14.9	67.0	52.6
Local governments	0.0	1.3	31.4	32.7	25.7

Source: Federal Highway Administration, 2005 (*Highway Statistics 2000*, Table HF-10).

TABLE 3: U.S. Transit Expenditures by Agency and Type, 2000

Expenditure Type and Funding Source	Expenditure (millions)	Percent
Capital expenditures		
Federal government	$6,354	54.3
State government	1,011	8.6
Local government	4,345	37.1
Subtotal	$11,710	100.0
Operating expenditures		
Funded by passenger revenues	$8,115	35.9
Funded by federal government	$232	1.0
Funded by state government	$5,127	22.7
Funded by local government	$7,937	35.1
Funded by other funds	$1,204	5.3
Subtotal	$22,615	100.0
Total expenditures	$34,325	

Source: Federal Transit Administration, 2005 (*National Transit Database 2000*).

percent was covered by passenger revenues. The subsidies to the operation of public transportation were $7.9 billion, or 35 percent, from local governments, $5.1 billion, or 23 percent, from state governments, and 0.2 billion, or 1 percent, from the federal government. The remaining $1.2 billion, or 5 percent, was funded from other sources. Capital expenditures for public transit totaled $11.7 billion in 2000. Of this amount, the federal government contributed $6.4 billion, or 54 percent; state governments contributed $1.0 billion, or 9 percent; and local governments contributed $4.3 billion, or 37 percent. There is no allocation of passenger fares between capital and operating expenditures, but, as noted above, passenger fares amounted to only 23.6 percent of total operating and capital expenditures of $34.3 billion.

3. TRANSPORTATION SUBSIDIES IN A MONOCENTRIC URBAN AREA WITH TWO MODES

In this section, we present a two-mode urban model, which is a modification of the standard monocentric urban model (Brueckner, 1987). The comparative static results for the main endogenous variables are tabulated and discussed heuristically in the text. The appendix provides mathematical derivations. We do not assess the welfare implications of transport subsidies (see Sasaki, 1989), nor do we consider the political support for transport subsidies that may be engendered by land-rent changes due to subsidies (see Borck and Wrede, 2005; Borck and Wrede, forthcoming).

The monocentric model has been criticized on many grounds (see Anas, Arnott, and Small, 1998, pp. 1435–1436, for a brief summary). Its most obvious shortcoming, which also accounts for its name, is the assumption of a single center, the central business district (CBD), to which all residents commute for work and other activities. Even the most casual observer of urban areas can see that this assumption is clearly untrue. Why, then, do we use the monocentric model?

We use the monocentric model for theoretical and empirical reasons. Theoretically, there is a canonical monocentric theory (Brueckner, 1987), which is not the case for polycentricity. Section 5 of Anas, Arnott, and Small (1998, pp. 1444–1454) is entitled, "Theories of Agglomeration and Polycentricity," where *theories* refers to both agglomeration theories and polycentric theories, so there is no canonical polycentric theory.

In our opinion, Anas, along with his coauthors (Anas and Kim, 1996; Anas and Xu, 1999), has provided the best available polycentric theory, a theory that avoids most of the criticisms leveled at the monocentric model. Nevertheless, Anas (2007) leaves the empirical application of his theory as an extension. On the other hand, the monocentric model lends itself easily to empirical estimation and has proved robust. Estimated coefficients are mostly statistically significant and have the theoretically predicted signs in samples ranging from relatively small urbanized areas (Brueckner and Fansler, 1983) to those including almost all urbanized areas in cross-section and multi-year pooled cross-section regressions (McGrath, 2005; Song and Zenou, 2006).

We assume two modes of transportation, with total transportation costs given by the following equation:

(1) $$M_i = f_i + (1 - \alpha_i)t_i x \quad i = 1, 2,$$

where M_i is mode i's total cost; f_i is mode i's total fixed cost, which includes all travel costs independent of distance traveled; t_i is the marginal cost of travel, so $t_i x$ is mode i's total variable cost, which includes all costs that vary with distance traveled, x; and $(1 - \alpha_i)t_i x$ is that portion of mode i's total variable cost paid by the traveler, where α_i is the subsidized proportion of mode i's total variable cost $(0 < \alpha_i < 1)$. We assume the subsidy relates only to variable, not total, travel cost. This latter assumption somewhat simplifies the analysis while not detracting from the results. Finally, we assume $f_1 < f_2$ and $(1 - \alpha_1)t_1 > (1 - \alpha_2)t_2$, so that one mode will not dominate in the urban area. Travelers choose the mode that minimizes their transportation cost. Thus, travelers use mode 1 for trips less than $\hat{x}$ and mode 2 for trips greater than $\hat{x}$, where $\hat{x} = (f_2 - f_1)/[(1 - \alpha_1)t_1 - (1 - \alpha_2)t_2)]$. Others who have used this or a similar formulation for modal transportation costs include Anas and Moses (1979), LeRoy and Sonstalie (1983), and Sasaki (1989, 1990).

Although the theory defines modes only in terms of their fixed and variable costs, we think of mode 1 as public transit and mode 2 as automobile transportation. Public transit has a lower fixed cost than the automobile but a higher marginal (mainly time) cost. Also, public transit is generally the mode used by those living closer to the CBD, while the automobile is generally used by those living farther out. In the empirical analysis, mode 1 is bus transport, and mode 2 is auto transport.[2]

A household's quasi-concave utility function is

(2) $$v = v(c, q),$$

where q is land consumption, which is a normal good, and c is nonland, nontransportation expenditures.[3] The household has the budget constraint

(3) $$y = \theta y + c + rq + M_i,$$

which says the household spends its exogenous income, y, on taxes, θy, where θ is the "income" tax rate $(0 < \theta < 1)$; nonland, nontransportation goods, c; land, q, where r is the rent of land; and transportation, M_i. The tax rate, θ, is meant to capture the combined property, sales, income, and other

[2]This approach is similar to that of LeRoy and Sonstelie (1983) and of Sasaki (1990), except that they explicitly include time costs as part of the variable costs of travel. We have subsumed time costs into fixed and marginal transport costs, as did Sasaki (1989). These authors also assume that auto fixed cost exceeds transit fixed cost and that transit variable cost exceeds auto variable cost, the latter due to time costs. Arnott and MacKinnon (1977) also distinguish money and time costs of travel and include mode choice in a simulation model. An extension of our model would be to include time costs explicitly, perhaps following the approach of Arnott and MacKinnon (1977), LeRoy and Sonstelie (1983), DeSalvo (1985), Sasaki (1990), or Krugman (1991).

[3]An earlier version of this paper included a housing sector. On the advice of referees for this journal, we have excluded that feature of the model, but see Su (2006) for a model including the housing sector.

taxes paid by an urban-area resident, expressed as a fraction of income. The term θy reflects the fact that an individual's tax payments tend to rise with income.

The problem of the household is to maximize equation (2) subject to equation (3). Upon eliminating c, this problem gives rise to the first-order condition,

$$\frac{v_q[(1-\theta)y - rq - M_i, q]}{v_c[(1-\theta)y - rq - M_i, q]} = r. \tag{4}$$

All urban households are assumed to be identical with respect to utility function and income. Consequently, for them to be in spatial equilibrium, in which no one wants to move, the following condition must hold

$$v[(1-\theta)y - rq - M_i, q] = u, \tag{5}$$

where u is the urban-area-wide spatial equilibrium utility level. The variable, c, plays no role in the analysis and is therefore ignored.

Equations (4) and (5) are solved simultaneously for r_i and q_i and are functions of the following variables: $\theta, y, f_i, \alpha_i, t_i, x$, and u, where the subscript i's refer to the subscripted variable's value in the vicinity of mode i.

The urban boundary conditions are

$$r_1(\hat{x}) = r_2(\hat{x}), \tag{6}$$

and

$$r_2(\bar{x}) = r_A, \tag{7}$$

where $\bar{x}$ is the distance from the CBD at which the urban area ends and the rural area begins and r_A is rural land rent. There are two boundary conditions because we need to distinguish the boundary between the two modes and the boundary between the urban and rural areas. The notation r_i denotes these function in the vicinity of mode i.

Equation (6) is not used to solve for $\hat{x}$, for that is obtained as noted above. Instead, equation (6) simply ensures the continuity of the land-rent function at $\hat{x}$. Equation (7), on the other hand, is used to solve for $\bar{x}$, as will be explained below.

The urban population condition is

$$\int_0^{\hat{x}} \frac{\delta x}{q_1} dx + \int_{\hat{x}}^{\bar{x}} \frac{\delta x}{q_2} dx = L, \tag{8}$$

where δ is the number of radians in a circle available for urban residential use, $1/q_i$ is population density in the area of mode i, and L is the urban population, which is assumed to be the same as the number of urban households. This condition ensures that the population of the urban area fits inside the boundary of the urban area.

In the closed city model, which we are using, u and $\bar{x}$ are obtained by solving equations (7) and (8) simultaneously and are functions of the following variables: $\theta, y, r_A, \delta, L, f_1, \alpha_1, t_1, f_2, \alpha_2$, and t_2.

In a closed city model, in which population is exogenous and the spatial equilibrium utility level is endogenous, transportation subsidies result in relocation of households within the urban area. If, for example, auto subsidies increase, some transit users switch to auto and the modal boundary decreases. For those already using auto, income net of transportation cost increases. Consequently, these households consume more housing and move farther from the CBD. Thus, the effect on both transit users and auto users of an increase in auto subsidies contributes to urban sprawl.

In the open city model, in which population is endogenous and the spatial equilibrium utility level is exogenous, transportation subsidies result in migration among areas. If, for example, auto subsidies increase in an urban area, disequilibrium spatial utility increases in that urban area, which attracts migration to the urban area. Increased population reduces the disequilibrium spatial utility level, and population growth ends when the exogenous spatial utility level is reestablished. The increase in population expands the spatial size of the urban area to meet the increased demand for housing.

Thus, the theoretical effect is qualitatively the same in both models. Empirically, however, population is a major determinant of the spatial size of an urban area (Brueckner and Fansler, 1983; McGrath, 2005; Song and Zenou, 2006). Thus, to eliminate population as a regressor, which one would have to do in the open city model, would eliminate a major source of urban spatial growth. Also, as far as we know, no other researchers have employed the open city model in empirical analysis.

Since municipalities and counties are required by state law to balance their budgets, we add the balanced-budget equation

$$\beta\theta yL + G = \alpha_1 \int_0^{\hat{x}} \frac{t_1 x^2}{q_1} dx + \alpha_2 \int_{\hat{x}}^{\bar{x}} \frac{t_2 x^2}{q_2} dx, \tag{9}$$

where the left-hand side is urban-area government revenues devoted to urban-area transportation and the right-hand side is urban-area government expenditures on transportation. The first term on the left-hand side is the share of urban-area tax revenues used for transportation, where β is the tax share ($0 < \beta < 1$), and G is intergovernmental grants devoted to transportation. The first term on the right-hand side is the share of urban-area public transit costs paid by the urban-area government, where α_1 is the proportion of total transportation expenditures subsidized by the urban-area government and the integral is the total cost of transportation via mode 1 in the urban area. The second term on the right-hand side is similarly defined for mode 2.

This formulation of the government budget constraint is guided by how highways and public transportation are financed in the U.S. As discussed in Section 2, much of the subsidy to urban transportation is paid by local governments from revenues generated by various taxes and other sources as well as by intergovernmental grants from the federal and state governments. If θ increases, tax revenues for local government increase. This, however, does not necessarily mean that the new revenue is used to finance transportation. In

TABLE 4: Comparative Static Results

Exogenous Variable	L	y	r_A	θ	G	f_1	t_1	α_1	f_2	t_2	α_2
Effect on $\bar{x}$	+	+	−	−	0	+	+	−	−	−	+

our model, how much local tax revenue is used as subsidies is determined by β. Our balanced budget condition simply reflects the fact that transportation finance is one of the key expenditures of local government, but not the only one. Our model implicitly assumes that local government pays transportation costs not covered by intergovernmental grants. In fact, after local governments determine how much they can get from federal and state governments for highway or transit expenditures, local government pays the rest (Government Accounting Standards Board, 2007). We think it more likely that transportation subsidies are funded at the expense of other public goods than by increasing local tax rates. If one examines a county's financial report, one sees that when there is a deficiency in the transportation fund, the county has to transfer money from other funds, usually from the general fund. Thus, more transportation subsidies mean smaller amounts of other public goods (Government Accounting Standards Board, 2007).

Finally, our formulation avoids making the "income" tax rate endogenous, which would considerably complicate the comparative static analysis. By making the tax-share variable a "slack" variable, we avoid impacts on other endogenous variables due to income effects on tax revenues through income, population, and the "income" tax rate. (Note that the budget-share variable does not appear in any other structural equation.)

Because equation (9) is a function of $\hat{x}$ and $\bar{x}$, the endogenous variable, β, is a function of the same variables as are $\hat{x}$ and $\bar{x}$ as well as of the variable unique to equation (9), namely, G. Thus, in general, β is a function of the following variables: $\theta, y, G, r_A, \delta, L, f_1, \alpha_1, t_1, f_2, \alpha_2$, and t_2.

Table 4 summarizes the comparative static results of changes in exogenous variables on the spatial size of the urban area. Here, we provide heuristic explanations for these results, while the Appendix contains the mathematical derivations.

Both this model and the standard monocentric urban model (Brueckner, 1987) find a direct effect of population and income and an inverse effect of rural land rent on the urban radius. Although there is no income tax in the standard model, since our tax rate is inversely related to disposable income, then its effect on the urban areas' spatial size is the reverse of the income effect. Remaining results either differ or are not comparable between the models.

The intergovernmental grant variable has no effect on the size of the urban area because, in our model, an increase in G is completely offset by a decrease in local tax revenues.

The standard model finds that the urban radius is inversely related to marginal transport cost, while our model finds a direct relationship for transit's (mode 1's) marginal transportation cost and an inverse relationship for auto's (mode 2's) marginal transportation cost. The standard model does not have variables for fixed cost of travel nor for transportation subsidies. However, since it is reasonable that the cost variables should operate inversely to the subsidy variables, to save space, we restrict our explanation to the subsidy effects, and it is these effects that the paper mainly investigates.

An increase in transit subsidies expands the area using transit ($\partial\hat{x}/\partial\alpha_1 > 0$). For those residents located in the area between the new $\hat{x}$ and the original $\hat{x}$, transit's commuting cost is now lower than auto's, and those residents switch from auto to transit. All transit users' incomes net of commuting cost rise, which bids up land rent between the CBD and the new modal boundary, thereby raising population density there. One would expect this effect to expand the urban area, but in our model income does not affect the modal boundary, so there is no urban expansion. Those remaining auto users beyond the new modal boundary find their incomes net of commuting cost decrease. This bids down land rent beyond the new modal boundary, thus lowering population density there and shrinking the size of the urban area.

An increase in auto subsidies expands the area using auto ($\partial\hat{x}/\partial\alpha_2 < 0$). For those residents located in the area between the new $\hat{x}$ and the old $\hat{x}$, auto's commuting cost is now lower than transit's, and those residents switch from transit to auto. All auto users' incomes net of commuting cost rise, which bids up land rent between the CBD and both the new modal boundary and the urban-rural boundary, so the urban area expands and population density rises beyond the new modal boundary. The remaining transit users find their incomes net of transportation subsidies unchanged, but the movement of population away from the CBD lowers land rent and population density there.

4. EMPIRICAL ANALYSIS

Data Description

The census-delineated urbanized area is the statistical analog to our theoretical urban area. For consistency with our monocentric assumption, of the 465 urbanized areas delineated in the 2000 census, we choose as our sample the 201 urbanized areas that have only one central city located within a single county (the sample size ultimately falls to 93 because of data availability). Our dependent variable is the spatial size of the urbanized area, *Area*, which is a proxy for the radius of the urban area, $\bar{x}$. In the theoretical model, population, L, and the number of households are equal because we assume single-person households. In the empirical work, L is the number of urbanized-area households. The model assumes everyone has the same income, y, so in the empirical work, we use mean household income. Mean household income is not reported by the census. We derive it by dividing aggregate household income, which is reported by the census, by the number of urbanized-area households. In the model, agricultural land rent, r_A, is land rent at the urban fringe. This theoretical construct is not readily available empirically. As an alternative, we use mean estimated market value of farmland per acre for the county in which the urbanized area is contained.

Our theoretical model does not specify the public transit mode. For our empirical work, we use bus as the public transit mode because bus service is the most widely provided public transit mode in the United States. Of the 518 transit agencies that reported fare revenues to the Federal Transit Administration in 2000 (Federal Transit Administration, 2005), 85 percent of the agencies provided this service, while the next most widely provided form of public transportation, light rail, was offered by only 4 percent of the agencies. Of the 201 urbanized areas in our sample, 111 provided bus service.

In the theoretical model, fixed transit cost, f_1, includes the value of travel time between home and transit stop as well as waiting time at the transit stop. There are, however, no data by urbanized area on these variables for bus transit. As a proxy for fixed transit cost, we use the percentage of the working-age population taking bus to work. We expect this variable to be inversely related to fixed transit cost because the longer the waiting time, the less attractive transit becomes as a means of transportation, and the fewer users it will attract. This assumption is supported by Zhao et al. (2002), who report that transit use deteriorates exponentially with walking distance to transit stops. Also, travel-behavior studies find that waiting time is more onerous than in-vehicle travel time (U.S. Department of Transportation, 1986).

Marginal transit cost, t_1, is the household's cost of transit per round-trip mile. For bus transportation, the American Public Transit Association (2002) reports adult single-trip base fares as well as charges for transfers and zone changes. We would like to use the single-trip fare as a proxy for marginal transit cost, but the variable has very small variation among transit agencies nationwide, which makes it unsuitable for our use. Instead, we use private transit cost per passenger-mile. We calculate marginal transit cost per passenger-mile by dividing annual bus fare revenue by annual total passenger miles per urbanized area, which are reported by the Federal Transit Administration (2005) for the year 2002. We use 2002 data instead of 2000 data because the national transit database first started reporting revenue and expenses by mode in 2002. Prior to 2002, transit agencies reported only total operating and capital expenditures. Since not all agencies operate the same modes and some operate several modes, if we use the aggregate data prior to 2002, we would be comparing apples and oranges. Nevertheless, we compared the aggregate totals of expenses and revenues for 2000 and 2002 (deflated to 2000 dollars), and found little difference, which increases our confidence in the 2002 data.

Theoretically, the transit subsidy, α_1, is the subsidized share of the household's round-trip transit cost per mile. Our proxy is bus subsidy per passenger-mile. To find the transit subsidy per commuter-mile, we divide the total transit subsidy (which we define as the difference between annual bus capital and operating cost and annual bus fare revenue) by annual total bus passenger-miles. The Federal Transit Administration (2006) provides these data by transit agencies for the year 2002. We derive the urbanized-area transit subsidies per passenger-mile by matching each urbanized area with the transit agencies serving that area.

In our theoretical model, auto fixed cost, f_2, includes all costs that are invariant to distance traveled. For highway users, fixed cost generally includes registration fees, license fees, motor vehicle taxes, auto insurance premiums, part of depreciation, and financial charges. Since cost data on depreciation and financial charges are not readily available, we use the sum of average annual motor vehicle fees and taxes and annual auto insurance premiums per household to represent fixed auto cost. These costs amount to approximately 25 percent of auto total fixed costs (American Automobile Association, 2006).

In our theoretical model, auto marginal cost, t_2, is the household's auto expenditure per round-trip mile. No readily available measure of this cost exists by urbanized area. Consequently, we use highway fuel tax payments per vehicle-mile traveled (VMT) to represent auto variable cost. These payments amount to approximately 28 percent of total auto variable costs (Energy Information Administration, 2006).

For consistency with our theoretical model, the highway subsidy, α_2, should be measured per round-trip mile per household for travel within an urban area. Highway subsidies are the difference between highway expenditures paid by various levels of government in a given urban area and the total user-fees collected from highway users in the same area. Such a measure of highway subsidies is unavailable because of complicated highway ownership and financing systems in the United States.

Highway financing is largely, but not solely, determined by highway ownership. In the United States, ownership is divided among federal, state, and local governments. According to the Federal Highway Administration (2003), states own almost 20 percent of the nation's road system. The federal government has control of about 3 percent of the network. Over 77 percent of U.S. roads are locally owned although some intergovernmental agreements may authorize states to construct and maintain locally owned highways. Highways owned by the federal government are 90 percent funded by the federal government. For highways owned by states, capital outlays are largely funded by the federal government through intergovernmental grants. Even local roads are not funded solely by local governments. State and federal governments fund the majority of capital outlays for local roads (Federal Highway Administration, 2003).

Given the complicated highway ownership and funding systems, it is almost impossible to obtain an accurate estimate of highway subsidies by urbanized area. For example, in a particular urbanized area, urban residents may have access to interstate highways owned and mainly funded by the federal government but maintained by the state government, state-owned highways funded by the state, highways owned by counties or cities maintained and funded by the state, and county or city roads owned and funded by those governments. For the various roads within an urbanized area, local governments report only revenues and expenditures on the roads owned and funded by them. No financing data are available for those roads not owned or funded by local governments. We are left therefore with county and city data to calculate highway subsidies. Unfortunately, however, city data are not reliable because most highway revenue data are not reported. We are therefore restricted to using only county data to calculate highway subsidies. As discussed in Section 2, local governments provide \$30.4 billion, or 63 percent of total subsidies to highway users. It is reasonable to believe that our proxy should be strongly correlated with the true variable, thus enabling us to capture the variation in highway subsidies among urbanized areas.

In the theoretical model, θ is the "income" tax rate, which is meant to capture income, sales, property, and other local taxes. We are unable to obtain income-tax and sales-tax data by urbanized area, so for the empirical analysis we use the property tax per household divided by mean household income. To estimate θ, we deflate 1997 and 2002 county and city property tax revenues to 2000 dollars by the CPI (U.S. Bureau of Labor Statistics, 2000). Our data are from the Census of Government (U.S. Bureau of the Census 2000c), which collects data every five years but in different years from the decennial years. To derive the per-household property tax payment, we average the property tax figures and divide by the number of urbanized-area households. Then the per-household property tax payment is divided by mean household income to derive the urban-area tax rate. Song and Zenou (2006) use GIS technology to obtain a weighted average of property tax rates to reflect interarea differences in property taxes and get results similar to ours.

Table 5 provides a summary of definitions and data sources for the variables used. Table 6 provides units in which the regression variables are measured and descriptive statistics for these variables.

Estimation Issues

We estimate the following equation:

$$\ln Area = \beta_0 + \beta_1 L + \beta_2 y + \beta_3 y^2 + \beta_4 r_A + \beta_5 \theta + \beta_6 G + \beta_7 f_1 + \beta_8 t_1 + \beta_9 \alpha_1 + \beta_{10} f_2 + \beta_{11} t_2 + \beta_{12} \alpha_2 + \beta_{13} \alpha_2^2 + \beta_{14} S + \varepsilon, \tag{10}$$

where, S, a state dummy variable, is used partially to account for the age of the city and underlying differences in state planning laws and other factors that may influence urban size.[4] We adopt the semilog form for the following reasons. Following Wooldridge (2006, pp. 218–220), we obtain a goodness-of-fit measure for the log model that can be compared with the R^2 from the level model. We find that the semi-log model explains more of the variation in the spatial size of urbanized areas than does the linear model. As suggested by a referee, we tested the natural logarithm of population as a regressor. The results were generally

[4]We experimented with the following variables to reflect city age: percentage of buildings built before 1940, 1950, or 1960; and median building age. None of them is statistically significant.

TABLE 5: List of Variables, Definitions, and Data Sources

Variable	Definition	Source
Area	Spatial size of the urbanized area in square miles	U.S. Bureau of the Census (2000b)
L	Number of households by urbanized area	U.S. Bureau of the Census (2000a)
r_A	Estimated market value of farmland per acre for the county in which the urbanized area is located	National Agricultural Statistics Service (1999, 2004)
y	Mean household income by urbanized area	U.S. Bureau of the Census (2000a)
f_1	Percentage of working age population using transit	U.S. Bureau of the Census (2000a)
f_2	Sum of auto annual insurance premium, registration fee, license fee, and motor vehicle tax per household by urbanized area	Federal Highway Adm. (2005); Insurance Information Institute (2005)
t_1	Bus fare cost per passenger-mile	Federal Transit Adm. (2006)
t_2	Fuel tax payment per vehicle-mile traveled	Federal Highway Adm. (2005)
α_1	Subsidies to bus service per passenger-mile	Federal Transit Adm. (2005)
α_2	County subsidies to auto use per vehicle-mile traveled	U.S. Bureau of the Census (2000c)
G	Intergovernmental transfers from state to local governments for transportation purposes	U.S. Bureau of the Census (2000c)
θ	Local "income" tax rate: percentage of average household income paid as property tax payment	U.S. Bureau of the Census (2000c)

TABLE 6: Units of Measurement and Descriptive Statistics

Variable	Unit	Mean	Standard Deviation	Range Minimum	Range Maximum
Area	Square miles	85.8	75.0	13.6	323.6
L	1,000's	79.8	92.9	17.9	518.6
y	\$1,000's	49.6	8.0	31.4	92.9
r_A	\$1,000's	2.2	1.1	0.3	5.9
θ	Percentage	1.6	0.7	0.4	3.3
G	\$1,000's	20,380	58,908	0.95	500,000
f_1	Percentage	1.8	1.5	0.4	7.6
t_1	Cents	15.8	10.4	1.7	75.1
α_1	Cents	112.6	80.7	17.3	430.6
f_2	\$1,000's	1.3	0.2	1.0	1.7
t_2	Cents	3.5	1.0	1.7	6.6
α_2	Cents	0.4	0.3	0.007	1.7

similar to those reported below, but there were fewer statistically significant variables.

We conduct specification tests that are embedded in Stata, the software we use to run the regression. An added-value plot reveals that there are significant nonlinearities in the data for income and the auto subsidy. We, therefore, add two quadratic terms for these variables. This is further justified by the Lagrange multiplier test for adding variables. The chi-square value obtained by running a regression of the residuals of the restricted model on all explanatory variables is 27.44, exceeding the 1 percent critical value of 9.21 for 2 degrees of freedom, which indicates that we should reject the restricted model.

Theoretically, all the independent variables in equation (10) are exogenous. Econometrically, however, one or more of our explanatory variables may be endogenous if it is correlated with the error term, in which case ordinary least square (OLS) produces inconsistent estimates. There are several potential candidates for endogenous explanatory variables, including the percentage of working age population using transit (a proxy for fixed transit cost), bus marginal cost, bus subsidies, auto marginal cost, auto subsidies, and the property tax rate (identified as endogenous by Song and Zenou (2006)). If these variables were correlated with any of the unmeasured spatial size determinants that are buried in the error term, then the resulting coefficient estimates in equation (10) would suffer from omitted-variable bias. This bias cannot be eliminated because the omitted variables, by definition, are not in our data set.

When potential endogeneity is involved, researchers generally introduce instrumental variables (IVs) into the regression and use two-stage least squares (2SLS) to correct the problem. Since the 2SLS estimators are less efficient than the OLS estimators when the explanatory variables are exogenous, it is important to perform endogeneity tests to determine if 2SLS is necessary.

Since the highway subsidy is the difference between highway user charges and highway expenditures, the auto marginal cost per VMT and highway subsidies per VMT may be simultaneously determined. In our empirical analysis, we use three variables as IVs for the auto marginal cost per VMT: (1) state gasoline tax per gallon, (2) urbanized-area freeway lane-miles, and (3) number of interstate highway rays in 1970s, a variable used by Baum-Snow (2007). These variables, together with the crime rate per 1,000 bus users, are used as IVs for highway subsidies per VMT. We use three IVs for the three potential endogenous bus-related variables: (1) the crime rate per 1,000 bus users, (2) adult single-trip base fare for bus, and (3) Federal Urban Area Formula Program funds per passenger-mile. For the potential endogenous property tax, we use state school aid per student, the IV used by Song and Zenou (2006). Our first-stage specification tests suggest that all the potential endogenous variables, except for fixed bus cost, seem to have suitable IVs. The three IVs for fixed bus cost are not jointly significant and are thus considered

TABLE 7: Instrumental Variables, Definitions, and Data Sources

Variable	Definition	Source
Adult basic bus fare	Adult single-trip basic bus fare, excluding transfer fee	American Public Transportation Association (2000)
Bus stop crime rate	Crime rate at bus stops per 1,000 bus users	Federal Transit Administration (2005)
Federal urbanized area formula program funds per passenger-mile	Federal urbanized area formula program fund divided by total number of passenger miles by urban area	Federal Transit Administration (2006)
State gasoline tax rate	State gasoline tax rate per gallon	Federal Highway Adm. (2005)
Freeway lane miles	Freeway lane miles by urban area	Federal Highway Adm. (2005)
Number of rays	A ray is defined as a segment of interstate highways that connects the CBD of the central city with the region outside the central city	Baum-Snow (2007)
State school aid per student	State Aid to schools divided by total number of school-age students	National Center for Education Statistics (2007)

weak instruments.[5] Table 7 provides definitions of and data sources for these IVs.

Following Wooldridge (2006, pp. 532–535), we estimate the reduced form for the six potential endogenous variables by regressing them on all exogenous variables (including those in the structural equation and the additional IVs

[5]We ran first-stage regressions to test whether or not the IV's were suitable. Our specification tests indicate the following: for t_1 (bus variable cost), the crime rate per 1,000 bus users, adult single-trip base fare for bus, and Federal Urban Area Formula Program funds per passenger-mile are jointly significant, and adult single-trip base fare is individually significant at the 0.01 level or better, suggesting that they are suitable instruments for this variable; for t_2 (auto variable cost), the state gasoline tax rate, freeway lane miles, and the number of rays are jointly significant, and the first two variables are individually significant at the 0.01 level or better, suggesting that they are suitable instruments for t_2; for θ (the property tax rate), state school aid per student is statistically significant at the 0.10 level or better ($t = 1.94$), suggesting this variable is a suitable instrument for θ; for α_1 (the transit subsidies per passenger-mile), all three IV's have the right signs but none of them is statistically significant; for α_2 (the auto subsidies per VMT), the state gasoline tax rate, freeway lane miles, the number of rays, and the crime rate per 1,000 bus users, have a jointly significant effect, and the crime rate per 1,000 bus users is statistically significant at the 0.05 level or better; for f_1 (the percentage of working age population using transit), the three IV's are not jointly significant.

discussed above). We then add the residuals into the structural equation and test for joint significance of the six residuals in an OLS regression, using an F-test. The $F_{6,74}$ we obtain is 1.09, while the 10-percent critical value for the F-test is 1.855. This finding suggests that the suspected variables do not significantly bias the OLS results. We also conduct an overidentification test to see whether the IVs are exogenous. Following Wooldridge (2006, pp. 532–535), we first estimate the structural equation by 2SLS and obtain the residuals. Next, we regress the residuals on all exogenous variables and obtain the R^2. Under the null hypothesis that all IVs are uncorrelated with the 2SLS residuals, $nR^2 \sim \chi^2_q$, where n is the sample size and q is the number of IVs from outside the model minus the total number of endogenous variables. Since nR^2 equals 2.66, which is lower than the 10-percent critical value of 2.71, we cannot reject the hypothesis that all IVs are not correlated with the error term. Based on these two tests, it is reasonable to believe the OLS estimates do not appear to be statistically biased. We, therefore, focus on discussing the OLS results.

Empirical Results

Table 8 presents the OLS results, and Table 9 provides the correlation matrix of explanatory variables.[6] To get a sense of the sensitivity of sprawl to our explanatory variables, we calculate elasticities of the urban area's spatial size with respect to the individual explanatory variables, and we calculate their individual effects on the spatial size of the urban area due to a 1 percent change in the variable (all evaluated at variable means). In Table 10, we show results only for those explanatory variables whose OLS coefficients are statistically significant at the 10 percent level or better.

The coefficient on population, L, is positive, as predicted, and statistically significant. A 1 percent increase in the number of households, which is about 800 households, produces an approximately 0.6 percent increase in the spatial size of an urban area. This increases the size of the urban area by about 0.5 square miles, or about 0.4 acres per household, which is slightly larger than the common suburban residential lot size of one-third acre.

We find that the spatial size of the urbanized area increases at a decreasing rate with income, y, with both coefficients being statistically significant. At the mean urbanized-area income of \$49,600, the result is positive, which is in line with the theoretical prediction of our model. When income is \$56,600 or higher, however, our result is negative. Based on our sample, 84 percent of areas have positive income effects. A 1 percent increase in income, which is \$496, produces an approximately 0.7 percent increase in the spatial size of an urban area.

The coefficient on agricultural land value, our proxy for r_A, is positive but not statistically significant. This sign is inconsistent with the theoretical prediction. The inconsistency may be due to the fact that the mean estimated

[6]The coefficients on state dummies are suppressed. Only three state dummies are statistically significant: Michigan, Texas, and Wisconsin.

TABLE 8: OLS Regression Results

Variable	Coefficient	Std. Error	P
L	0.0073	0.0006	0.000
y	0.1356	0.0268	0.000
y^2	−0.0012	0.0002	0.000
r_A	0.0269	0.0347	0.440
θ	−0.1102	0.0612	0.076
G	–1.94e–7	9.93e–7	0.844
f_1	−0.1055	0.0258	0.000
t_1	0.0093	0.0039	0.020
α_1	−0.0012	0.0005	0.016
f_2	−0.3957	0.2292	0.008
t_2	−0.0396	0.0390	0.314
α_2	0.9000	0.3563	0.014
α_2^2	−0.7779	0.2820	0.007
Constant	0.8992		
R^2	0.8365		
N	93		

TABLE 9: Correlation Matrix of Explanatory Variables

	L	r_A	y	θ	G	f_1	t_1	α_1	f_2	t_2	α_2
L	1.00										
r_A	0.09	1.00									
y	0.60	0.24	1.00								
θ	–0.11	–0.16	–0.0004	1.00							
G	0.59	–0.04	0.26	–0.06	1.00						
f_1	0.26	0.19	0.37	0.15	0.22	1.00					
t_1	0.03	0.04	–0.003	0.09	0.05	0.18	1.00				
α_1	–0.14	–0.004	–0.06	0.06	0.13	–0.18	0.38	1.00			
f_2	0.24	0.02	0.32	–0.21	0.20	0.12	–0.10	–0.14	1.00		
t_2	–0.18	–0.22	–0.18	0.17	0.08	–0.02	–0.10	0.01	–0.14	1.00	
α_2	0.13	–0.04	0.18	0.20	0.40	0.19	0.14	–0.07	0.19	0.18	1.00

TABLE 10: Elasticities and a Real Change Due to 1 Percent Change in Explanatory Variables

Variable	L	y	f_1	t_1	α_1	f_2	α_2	θ
Elasticity	0.59	0.82	–0.19	0.15	–0.14	–0.43	0.11	–0.18
Area Change	0.50	0.70	–0.17	0.13	–0.12	–0.50	0.10	–0.16

market values of rural land cannot capture actual rural land rent immediately adjacent to the built-up part of the urban area or from the variable's small range of variation in our sample, which may prevent the emergence of a precise estimate.

The coefficient on the property tax rate, θ, is negative, as predicted by our model, and statistically significant. Song and Zenou (2006) similarly find an inverse relation between the property tax and urbanized area size. A 1 percent increase in the property tax as a percentage of household income, our proxy for the urban-area "income" tax, produces a 0.18 percent decrease in urban-area size, or about 0.15 square miles. Our property tax elasticity is smaller than that obtained by Song and Zenou (0.401). This may result from the fact that we only count household property tax payments to the county and the city, which are roughly 30 percent of total household property tax payments.

The coefficient on the intergovernmental grant for highway purposes, G, is negative but not statistically significant. This result is consistent with our theoretical model, in which there is no effect of intergovernmental grants on the spatial size of an urban area.

The coefficient on our proxy for bus fixed cost, f_1, the percentage of people using bus, is negative, as predicted, and statistically significant. A 1 percent increase in the percentage of commuters using bus, which is an increase of about 0.02 percent, reduces the urban area by about 0.2 percent, or by about 0.2 square miles. The percentage of commuters using bus is a proxy for our theoretical variable, and we expect it to be inversely related to the theory's public transit fixed cost variable. Therefore, an increase in bus fixed cost increases urban-area size, which is consistent with our theory. This result indicates that the higher the percentage of commuters using transit, the smaller the urban area, other things equal.

The coefficient on bus marginal cost, t_1, is positive, as expected, and statistically significant. A 1 percent increase in bus cost per passenger-mile, which is about 0.01 cent, increases urban-area size by about 0.15 percent, or by slightly more than 0.1 square miles. This result suggests that the higher the transit cost, the larger the urban area, which is consistent with our theoretical prediction. The coefficient on bus subsidy, α_1, is negative and statistically significant, suggesting that the higher the bus subsidy, the smaller is the urbanized area. A 1 percent increase in the bus subsidy per passenger-mile, α_1, which is about \$0.01, reduces urban-area size by about 0.1 percent, or about 0.1 square mile.

The coefficient of auto fixed cost, f_2, is negative and statistically significant. This is consistent with our theoretical prediction. A 1 percent increase in auto fixed cost, about \$13 based on our sample, decreases urban-area size by 0.5 percent, or about 0.4 square miles. This result indicates that the higher the auto fixed cost, the smaller the area, other things equal.

The coefficient on auto marginal cost, t_2, is negative, as predicted, but not statistically significant.[7] This may be due to the fact that our measure represents only about 25 percent of auto variable cost.

[7]We experimented with an alternative measure of auto variable cost, weighted county transportation expenditures per person driving to work, which is used by Song and Zenou (2006). This measure is not suitable, however, because of its high correlation with highway subsidies.

The coefficient on auto subsidy, α_2, is positive and statistically significant, while that on auto subsidy squared is negative and statistically significant, indicting that the spatial size of the urbanized area is increasing at a decreasing rate with respect to the highway subsidy. At its mean value, the auto subsidy increases the urbanized area's spatial size, which is consistent with our theoretical prediction. However, when the auto subsidy per VMT is 0.5785 cents or higher, its effect is negative. About 80 percent of the areas in our sample have a positive auto subsidy effect. A 1 percent increase in county highway subsidies per VMT, an increase of about 0.004 cents, increases urban-area size by about 0.1 percent, or about 0.1 square miles.

In summary, our results show that the spatial size of an urban area is inversely related to transit subsidies and directly related to auto subsidies (at a decreasing rate for the latter). Although other measures of urban sprawl have been proposed (Newman and Kenworthy, 1998; McDonald, 1989; Malpezzi 1999; Glaeser and Kahn, 2004), our measure is theoretically and empirically supported by our findings.[8]

5. CONCLUSION

This paper investigates the effect of transit subsidies and highway subsidies on urban sprawl through both theoretical and empirical analyses. The theoretical model incorporates two modes in an urban spatial model, and our comparative static analysis indicates that the urban area shrinks with an increase in transit subsidies but expands with an increase in auto subsidies. In the empirical analysis, we estimate coefficients of a regression equation relating an urban area's size to bus subsidies, auto subsidies, population, mean income, rural land value, fixed and variable bus and auto costs, and the property tax rate. Our empirical results regarding the effect of transportation subsidies on urban sprawl are that subsidization of public transit reduces urban sprawl, while subsidization of automobile travel contributes to urban expansion, but at a decreasing rate.

If curbing urban sprawl is desirable, then certain policies emerge from our empirical analysis. We begin with public transit. Our finding on bus fixed cost suggests that policies to reduce waiting time would attract more people to bus use, which would help curb urban sprawl. Similarly, reducing bus fares would stimulate bus usage, which would also help curb sprawl. These are both consistent with our finding that higher subsidization of bus would lead to less sprawl. Variables affecting bus use, however, have relatively small effects on

[8]Among the density variables used to measure urban sprawl, we have data only on average population density, given our sample of small and mid-sized urbanized areas. Since the higher the population density, the lower the sprawl, we regress the natural log of inverse population density on our regressors. We find that auto subsidies, the percentage of commuters using transit (our proxy for fixed transit cost), and the property tax rate have statistically significantly positive effects on urban sprawl. Other variables have the right signs, but are only jointly statistically significant.

the spatial size of urban areas. This is to be expected since most of the urban areas in our sample are small and mid-sized areas where transit use is below the national average. The findings nevertheless suggest that increasing transit usage would help curb sprawl. Transit capacity is far from fully used in most urban areas. On average, a vehicle with a capacity of 40 passengers serves only 11 passengers (Federal Highway Administration, 2004). This suggests that increasing transit ridership may be very cost-effective since the marginal cost of serving additional passengers is low.

Turning to auto travel, our finding on auto fixed cost suggests that increasing auto fixed costs—such as registration fees, license fees, motor vehicle taxes, and parking fees—would help curb urban sprawl. Although our finding on auto marginal cost is statistically insignificant, it is likely that raising that cost would also help curb sprawl. These results are consistent with our finding on auto subsidization. Mills (1999), among others, has called for considerably higher taxes on gasoline as a way of reducing automobile use. Many industrialized countries in Europe and Asia have already adopted policies to reduce auto use. For example, owning and operating a car in Japan entails considerable expense, including mandatory auto inspection every two or three years, with an average cost of \$1,000 to \$1,200 per inspection; various taxes; and high parking cost in cities and at workplaces. As a result, most Japanese urban residents use public transportation for their daily commuting (Japanese Automobile Manufacturing Association, 2007).

Finally, because our empirical analysis finds that the spatial size of an urban area is not very responsive to any single transportation cost or subsidy variable, we conclude that policy makers at all governmental levels should address urban sprawl through an integrated strategy.

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APPENDIX

Comparative Static Results for Land Rent and Land Consumption

The land-rent function and land-consumption function are obtained from the simultaneous solution of equations (4) and (5) and depend on θ, y, f_i, α_i, t_i, x, and u. To obtain the comparative static effects of these variables on land rent and land consumption, we totally differentiate equation (5), getting

$$v_c[(1-\theta)dy - yd\theta - rdq - qdr - df_i - (1-\alpha_i)t_i dx \\ -(1-\alpha_i)xdt_i + t_i x d\alpha_i] + v_q dq = du.$$

Substituting $v_c r = v_q$ from equation (4) into the above equation and rearranging terms produces

(A.1)
$$-v_c q dr = du - v_c(1-\theta)dy + v_c y d\theta + v_c df_i + v_c(1-\alpha_i)t_i dx + v_c(1-\alpha_i)xdt_i - v_c t_i x d\alpha_i.$$

Then the following effects of changes in exogenous variables on r may be derived from equation (A.1)

(A.2)
$$\frac{\partial r_i}{\partial u} = -\frac{1}{v_c q_i} < 0,\ \frac{\partial r_i}{\partial x} = -\frac{(1-\alpha_i)t_i}{q_i} < 0,\ \frac{\partial r_i}{\partial y} = \frac{1-\theta}{q_i} > 0,\ \frac{\partial r_i}{\partial \theta} = -\frac{y}{q_i} < 0,$$
$$\frac{\partial r_i}{\partial f_i} = -\frac{1}{q_i} < 0,\ \frac{\partial r_i}{\partial \alpha_i} = \frac{xt_i}{q_i} > 0,\ \frac{\partial r_i}{\partial t_i} = -\frac{(1-\alpha_i)x}{q_i} < 0.$$

Except for changes in u, the effect of exogenous variables on q_i is given by the Hicksian demand slope, $(\partial q_i/\partial r_i)_u < 0$, times the price effects in equation (A.2) since u is held constant for these effects; so we have

(A.3)
$$\frac{\partial q_i}{\partial x} = \left(\frac{\partial q_i}{\partial r_i}\right)_u \frac{\partial r_i}{\partial x} > 0,\ \frac{\partial q_i}{\partial y} = \left(\frac{\partial q_i}{\partial r_i}\right)_u \frac{\partial r_i}{\partial y} < 0,\ \frac{\partial q_i}{\partial \theta} = \left(\frac{\partial q_i}{\partial r_i}\right)_u \frac{\partial r_i}{\partial \theta} > 0,$$
$$\frac{\partial q_i}{\partial f_i} = \left(\frac{\partial q_i}{\partial r_i}\right)_u \frac{\partial r_i}{\partial f_i} > 0,\ \frac{\partial q_i}{\partial \alpha_i} = \left(\frac{\partial q_i}{\partial r_i}\right)_u \frac{\partial r_i}{\partial \alpha_i} < 0,\ \frac{\partial q_i}{\partial t_i} = \left(\frac{\partial q_i}{\partial r_i}\right)_u \frac{\partial r_i}{\partial t_i} > 0.$$

Brueckner (1987, p. 825, n. 6) provides the effect of the spatial utility level on housing (in our case, land) consumption, which carries over to our model with appropriate notational changes. Therefore, we have

(A.4)
$$\frac{\partial q_i}{\partial u} = \left(\frac{\partial r_i}{\partial u} - \frac{\partial MRS_{qc}}{\partial c}\frac{1}{v_c}\right)\left(\frac{\partial q_i}{\partial r_i}\right)_u > 0,$$

since $\partial r_i/\partial u < 0$ from equation (A.2), $\partial MRS_{qc}/\partial c > 0$ to ensure that q is a normal good, and $v_c > 0$ by assumption.

Preliminary Results for Comparative Statics of Spatial Utility Level (u) and Urban-Rural Boundary ($\bar{x}$)

To obtain u and $\bar{x}$requires the simultaneous solution of equations (7) and (8), and the solutions depend on $\theta, y, r_A, L, f_1, \alpha_1, t_1, f_2, \alpha_2$, and t_2 (suppressing δ). To obtain the comparative static results, we totally differentiate equations (7) and (8) with respect λ, where λ stands for the exogenous variables listed above. We obtain an expression for $\partial u/\partial\lambda$ from equation (7) and an expression for $\partial\bar{x}/\partial\lambda$ from equation (8). We substitute the former into the latter to get the final result. We derive this equation below in general form. By making appropriate substitutions, we convert the equation to forms that yield the effect on $\bar{x}$ of specific exogenous variables. The only comparative static effect for which this approach does not work is $\partial \bar{x}/\partial G$.

Deriving $\partial u/\partial \lambda$ from equation (7). Totally differentiating equation (7) with respect to λ produces:

$$\frac{\partial \bar{r}_2}{\partial \lambda} + \frac{\partial \bar{r}_2}{\partial x}\frac{\partial \bar{x}}{\partial \lambda} + \frac{\partial \bar{r}_2}{\partial u}\frac{\partial u}{\partial \lambda} = \frac{\partial r_A}{\partial \lambda},$$

where the bar on a variable means that variable is evaluated at $\bar{x}$. From this expression, we obtain

$$\text{(A.5)} \qquad \frac{\partial u}{\partial \lambda} = \left[\frac{\partial r_A}{\partial \lambda} - \left(\frac{\partial \bar{r}_2}{\partial \lambda} + \frac{\partial \bar{r}_2}{\partial x}\frac{\partial \bar{x}}{\partial \lambda}\right)\right] \Big/ \frac{\partial \bar{r}_2}{\partial u}.$$

Deriving $\partial \bar{x}/\partial \lambda$ from equations (7) and (8). Rewrite equation (8) as

$$\text{(A.6)} \qquad \int_0^{\hat{x}} \frac{x}{q_1} dx + \int_{\hat{x}}^{\bar{x}} \frac{x}{q_2} dx - \frac{L}{\delta} = 0.$$

Totally differentiate equation (A.6) with respect to λ and solve for $\partial \bar{x}/\partial \lambda$, getting

$$\text{(A.7)} \quad \frac{\partial \bar{x}}{\partial \lambda} = \frac{\bar{q}_2}{\bar{x}}\left[\int_0^{\hat{x}} \frac{x}{q_1^2}\left(\frac{\partial q_1}{\partial \lambda} + \frac{\partial q_1}{\partial u}\frac{\partial u}{\partial \lambda}\right) dx + \int_{\hat{x}}^{\bar{x}} \frac{x}{q_2^2}\left(\frac{\partial q_2}{\partial \lambda} + \frac{\partial q_2}{\partial u}\frac{\partial u}{\partial \lambda}\right) dx + \frac{1}{\delta}\frac{\partial L}{\partial \lambda}\right],$$

where $\partial \delta/\partial \lambda = 0$ and where terms involving $\partial \hat{x}/\partial \lambda$ cancel out because $\hat{q}_1 = \hat{q}_2$. At $\hat{x}$, $M_1 = M_2$ and $r_1 = r_2$. Thus, the budget lines, $c_i = [(1-\theta)y - M_i] - r_i q_i$ ($i = 1,2$), have the same c-intercept (i.e., $(1-\theta)y - M_1 = (1-\theta)y - M_2$) and the same slope (i.e., $-r_1 = -r_2$). Because of spatial equilibrium, all households have the same utility level. Hence $\hat{q}_1 = \hat{q}_2$.

Substitute equation (A.5) into equation (A.7) and solve for $\partial \bar{x}/\partial \lambda$, getting

$$\text{(A.8)} \quad \frac{\partial \bar{x}}{\partial \lambda} = \frac{\displaystyle\int_0^{\hat{x}} \frac{x}{q_1^2}\left[\frac{\partial q_1}{\partial \lambda}\frac{\partial \bar{r}_2}{\partial u} + \frac{\partial q_1}{\partial u}\left(\frac{\partial r_A}{\partial \lambda} - \frac{\partial \bar{r}_2}{\partial \lambda}\right)\right] dx + \int_{\hat{x}}^{\bar{x}} \frac{x}{q_2^2}\left[\frac{\partial q_2}{\partial \lambda}\frac{\partial \bar{r}_2}{\partial u} + \frac{\partial q_2}{\partial u}\left(\frac{\partial r_A}{\partial \lambda} - \frac{\partial \bar{r}_2}{\partial \lambda}\right)\right] dx + \frac{1}{\delta}\frac{\partial L}{\partial \lambda}\frac{\partial \bar{r}_2}{\partial u}}{\displaystyle\frac{\bar{x}}{\bar{q}_2}\frac{\partial \bar{r}_2}{\partial u} + \int_0^{\hat{x}} \frac{x}{q_1^2}\frac{\partial q_1}{\partial u}\frac{\partial \bar{r}_2}{\partial x} dx + \int_{\hat{x}}^{\bar{x}} \frac{x}{q_2^2}\frac{\partial q_2}{\partial u}\frac{\partial \bar{r}_2}{\partial x} dx}.$$

The denominator in equation (A.8) is negative because $\partial \bar{r}_i/\partial u < 0$, $\partial q_i/\partial u > 0$, and $\partial \bar{r}_2/\partial x < 0$ from equations (A.2) and (A.4), so the sign of $\partial \bar{x}/\partial \lambda$ is opposite that of the numerator of equation (A.8).

Specific Comparative Static Results for $\partial \bar{x}/\partial \lambda$

Obtaining $\partial \bar{x}/\partial L$. Substitute $\lambda = L$ into equation (A.8), which becomes

$$\frac{\partial \bar{x}}{\partial L} = \frac{\displaystyle\frac{1}{\delta}\frac{\partial \bar{r}_2}{\partial u}}{\displaystyle\frac{\bar{x}}{\bar{q}_2}\frac{\partial \bar{r}_2}{\partial u} + \int_0^{\hat{x}} \frac{x}{q_1^2}\frac{\partial q_1}{\partial u}\frac{\partial \bar{r}_2}{\partial x} dx + \int_{\hat{x}}^{\bar{x}} \frac{x}{q_2^2}\frac{\partial q_2}{\partial u}\frac{\partial \bar{r}_2}{\partial x} dx} > 0,$$

because $\partial q_i/\partial L = \partial r_A/\partial L = \partial \bar{r}_2/\partial L = 0$ and because $\partial \bar{r}_2/\partial u < 0$.

Obtaining $\partial\bar{x}/\partial y$. Substitute $\lambda = y$ into equation (A.8), which becomes

$$\text{(A.9)}\quad \frac{\partial\bar{x}}{\partial y} = \frac{\displaystyle\int_0^{\hat{x}} \frac{x}{q_1^2}\left(\frac{\partial q_1}{\partial y}\frac{\partial\bar{r}_2}{\partial u} - \frac{\partial q_1}{\partial u}\frac{\partial\bar{r}_2}{\partial y}\right)dx + \int_{\hat{x}}^{\bar{x}} \frac{x}{q_2^2}\left(\frac{\partial q_2}{\partial y}\frac{\partial\bar{r}_2}{\partial u} - \frac{\partial q_2}{\partial u}\frac{\partial\bar{r}_2}{\partial y}\right)dx}{\displaystyle\frac{\bar{x}}{\bar{q}_2}\frac{\partial\bar{r}_2}{\partial u} + \int_0^{\hat{x}} \frac{x}{q_1^2}\frac{\partial q_1}{\partial u}\frac{\partial\bar{r}_2}{\partial x}dx + \int_{\hat{x}}^{\bar{x}} \frac{x}{q_2^2}\frac{\partial q_2}{\partial u}\frac{\partial\bar{r}_2}{\partial x}dx},$$

because $\partial r_A/\partial y = \partial L/\partial y = 0$. The sign of the numerator is ambiguous as it stands. Substituting from equations (A.2), (A.3), and (A.4) into the numerator of equation (A.9), after some manipulation, produces

$$\int_0^{\hat{x}} x\frac{1-\theta}{q_1^3\bar{q}_2}\left(\frac{\partial q_1}{\partial r_1}\right)_u\left(\frac{1}{v_c} - \frac{1}{\bar{v}_c}\right)dx + \int_0^{\hat{x}} x\frac{1-\theta}{q_1^2\bar{q}_2 v_c}\frac{\partial MRS_{qc}}{\partial c}\left(\frac{\partial q_1}{\partial r_1}\right)_u dx$$
$$+ \int_{\hat{x}}^{\bar{x}} x\frac{1-\theta}{q_2^3\bar{q}_2}\left(\frac{\partial q_2}{\partial r_2}\right)_u\left(\frac{1}{v_c} - \frac{1}{\bar{v}_c}\right)dx + \int_{\hat{x}}^{\bar{x}} x\frac{1-\theta}{q_2^2\bar{q}_2 v_c}\frac{\partial MRS_{qc}}{\partial c}\left(\frac{\partial q_2}{\partial r_2}\right)_u dx < 0,$$

because $(\partial q_i/\partial r_i)_u < 0$, $\partial MRS_{qc}/\partial c > 0$, and $\partial v_c/\partial x > 0$ (for a proof, see Wheaton (1974), p. 227). The latter inequality ensures $[(1/v_c) - (1/\bar{v}_c)] > 0$ because $\bar{v}_c > v_c$, where $\bar{v}_c$ is evaluated at $\bar{x}$ and v_c holds for $x < \bar{x}$. Thus, since the denominator and numerator of equation (A.9) are negative, we have $\partial\bar{x}/\partial y > 0$.

Obtaining $\partial\bar{x}/\partial r_A$. Substitute $\lambda = r_A$ into equation (A.8), which becomes

$$\frac{\partial\bar{x}}{\partial r_A} = \frac{\displaystyle\int_0^{\hat{x}} \frac{x}{q_1^2}\frac{\partial q_1}{\partial u}dx + \int_{\hat{x}}^{\bar{x}} \frac{x}{q_2^2}\frac{\partial q_2}{\partial u}dx}{\displaystyle\frac{\bar{x}}{\bar{q}_2}\frac{\partial\bar{r}_2}{\partial u} + \int_0^{\hat{x}} \frac{x}{q_1^2}\frac{\partial q_1}{\partial u}\frac{\partial\bar{r}_2}{\partial x}dx + \int_{\hat{x}}^{\bar{x}} \frac{x}{q_2^2}\frac{\partial q_2}{\partial u}\frac{\partial\bar{r}_2}{\partial x}dx} < 0,$$

because $\partial q_i/\partial r_A = \partial L/\partial r_A = \partial\bar{r}_2/\partial r_A = 0$ and because $\partial q_i/\partial u > 0$.

Obtaining $\partial\bar{x}/\partial\theta$. Substitute $\lambda = \theta$ into equation (A.8), which becomes

$$\text{(A.10)}\quad \frac{\partial\bar{x}}{\partial\theta} = \frac{\displaystyle\int_0^{\hat{x}} \frac{x}{q_1^2}\left(\frac{\partial q_1}{\partial\theta}\frac{\partial\bar{r}_2}{\partial u} - \frac{\partial q_1}{\partial u}\frac{\partial\bar{r}_2}{\partial\theta}\right)dx + \int_{\hat{x}}^{\bar{x}} \frac{x}{q_2^2}\left(\frac{\partial q_2}{\partial\theta}\frac{\partial\bar{r}_2}{\partial u} - \frac{\partial q_2}{\partial u}\frac{\partial\bar{r}_2}{\partial\theta}\right)dx}{\displaystyle\frac{\bar{x}}{\bar{q}_2}\frac{\partial\bar{r}_2}{\partial u} + \int_0^{\hat{x}} \frac{x}{q_1^2}\frac{\partial q_1}{\partial u}\frac{\partial\bar{r}_2}{\partial x}dx + \int_{\hat{x}}^{\bar{x}} \frac{x}{q_2^2}\frac{\partial q_2}{\partial u}\frac{\partial\bar{r}_2}{\partial x}dx},$$

since $\partial r_A/\partial\theta = \partial L/\partial\theta = 0$. The sign of the numerator is ambiguous as it stands. Substituting from equations (A.2), (A.3), and (A.4) into the numerator of equation (A.10), after some manipulation, produces

$$-\int_0^{\hat{x}} x\frac{y}{q_1^3\bar{q}_2}\left(\frac{\partial q_1}{\partial r_1}\right)_u\left(\frac{1}{v_c} - \frac{1}{\bar{v}_c}\right)dx - \int_0^{\hat{x}} x\frac{y}{q_1^2\bar{q}_2 v_c}\frac{\partial MRS_{qc}}{\partial c}\left(\frac{\partial q_1}{\partial r_1}\right)_u dx$$
$$-\int_{\hat{x}}^{\bar{x}} x\frac{y}{q_2^3\bar{q}_2}\left(\frac{\partial q_2}{\partial r_2}\right)_u\left(\frac{1}{v_c} - \frac{1}{\bar{v}_c}\right)dx - \int_{\hat{x}}^{\bar{x}} x\frac{y}{q_2^2\bar{q}_2 v_c}\frac{\partial MRS_{qc}}{\partial c}\left(\frac{\partial q_2}{\partial r_2}\right)_u dx > 0,$$

because $(\partial q_i/\partial r_i)_u < 0$, $\partial MRS_{qc}/\partial c > 0$, and $\partial v_c/\partial x > 0$. Thus, since the numerator is positive and the denominator is negative in equation (A.10), we have $\partial\bar{x}/\partial\theta < 0$

Obtaining $\partial \bar{x}/\partial G$. Totally differentiating equation (9) with respect to G, we have

$$\frac{\partial \beta}{\partial G} = -\frac{1}{\theta y L} < 0,$$

which means that when G increases, β falls sufficiently so that tax revenues are completely replaced by intergovernmental grants, and there are no changes in other endogenous variables. so $\partial \bar{x}/\partial G = 0$. Although equation (9) plays no role in determining changes in the urban-rural boundary, it is useful in determining $\partial\beta/\partial\lambda$, which we do not develop in this paper, but see Su (2006).

Obtaining $\partial \bar{x}/\partial f_1$. Substitute $\lambda = f_1$ into equation (A.8), which becomes

$$\frac{\partial \bar{x}}{\partial f_1} = \frac{\int_0^{\hat{x}} \frac{x}{q_1^2}\frac{\partial q_1}{\partial f_1}\frac{\partial \bar{r}_2}{\partial u}dx}{\frac{\bar{x}}{\bar{q}_2}\frac{\partial \bar{r}_2}{\partial u} + \int_0^{\hat{x}} \frac{x}{q_1^2}\frac{\partial q_1}{\partial u}\frac{\partial \bar{r}_2}{\partial x}dx + \int_{\hat{x}}^{\bar{x}} \frac{x}{q_2^2}\frac{\partial q_2}{\partial u}\frac{\partial \bar{r}_2}{\partial x}dx} > 0,$$

since $\partial r_A/\partial f_1 = \partial \bar{r}_2/\partial f_1 = \partial q_2/\partial f_1 = \partial L/\partial f_1 = 0$, $\partial q_1/\partial f_1 > 0$, and $\partial \bar{r}_2/\partial u < 0$.

Obtaining $\partial \bar{x}/\partial t_1$. Substitute $\lambda = t_1$ into equation (A.8), which becomes

$$\frac{\partial \bar{x}}{\partial t_1} = \frac{\int_0^{\hat{x}} \frac{x}{q_1^2}\frac{\partial q_1}{\partial t_1}\frac{\partial \bar{r}_2}{\partial u}dx}{\frac{\bar{x}}{\bar{q}_2}\frac{\partial \bar{r}_2}{\partial u} + \int_0^{\hat{x}} \frac{x}{q_1^2}\frac{\partial q_1}{\partial u}\frac{\partial \bar{r}_2}{\partial x}dx + \int_{\hat{x}}^{\bar{x}} \frac{x}{q_2^2}\frac{\partial q_2}{\partial u}\frac{\partial \bar{r}_2}{\partial x}dx} > 0,$$

since $\partial r_A/\partial t_1 = \partial \bar{r}_2/\partial t_1 = \partial q_2/\partial t_1 = \partial L/\partial t_1 = 0$, $\partial q_1/\partial t_1 > 0$, and $\partial \bar{r}_2/\partial u < 0$.

Obtaining $\partial \bar{x}/\partial \alpha_1$. Substitute $\lambda = \alpha_1$ into equation (A.8), which becomes

$$\frac{\partial \bar{x}}{\partial \alpha_1} = \frac{\int_0^{\hat{x}} \frac{x}{q_1^2}\frac{\partial q_1}{\partial \alpha_1}\frac{\partial \bar{r}_2}{\partial u}dx}{\frac{\bar{x}}{\bar{q}_2}\frac{\partial \bar{r}_2}{\partial u} + \int_0^{\hat{x}} \frac{x}{q_1^2}\frac{\partial q_1}{\partial u}\frac{\partial \bar{r}_2}{\partial x}dx + \int_{\hat{x}}^{\bar{x}} \frac{x}{q_2^2}\frac{\partial q_2}{\partial u}\frac{\partial \bar{r}_2}{\partial x}dx} < 0,$$

since $\partial r_A/\partial \alpha_1 = \partial \bar{r}_2/\partial \alpha_1 = \partial L/\partial \alpha_1 = 0$, $\partial q_2/\partial \alpha_1 = 0$, and $\partial \bar{r}_2/\partial u < 0$.

Obtaining $\partial \bar{x}/\partial f_2$. Substitute $\lambda = f_2$ into equation (A.8), which becomes

$$\text{(A.11)} \qquad \frac{\partial \bar{x}}{\partial f_2} = \frac{-\int_0^{\hat{x}} \frac{x}{q_1^2}\frac{\partial q_1}{\partial u}\frac{\partial \bar{r}_2}{\partial f_2}dx + \int_{\hat{x}}^{\bar{x}} \frac{x}{q_2^2}\left(\frac{\partial q_2}{\partial f_2}\frac{\partial \bar{r}_2}{\partial u} - \frac{\partial q_2}{\partial u}\frac{\partial \bar{r}_2}{\partial f_2}\right)dx}{\frac{\bar{x}}{\bar{q}_2}\frac{\partial \bar{r}_2}{\partial u} + \int_0^{\hat{x}} \frac{x}{q_1^2}\frac{\partial q_1}{\partial u}\frac{\partial \bar{r}_2}{\partial x}dx + \int_{\hat{x}}^{\bar{x}} \frac{x}{q_2^2}\frac{\partial q_2}{\partial u}\frac{\partial \bar{r}_2}{\partial x}dx},$$

since $\partial q_1/\partial f_2 = \partial r_A/\partial f_2 = \partial L/\partial f_2 = 0$. The sign of the numerator is ambiguous as it stands. Substituting from equations (A.2), (A.3), and (A.4) into the numerator of equation (A.11), after some manipulation, produces

$$-\int_0^{\hat{x}} x\frac{1}{q_1^3\bar{q}_2 v_c}\left(\frac{\partial q_1}{\partial r_1}\right)_u dx - \int_0^{\hat{x}} x\frac{1}{q_1^2\bar{q}_2 v_c}\frac{\partial MRS_{qc}}{\partial c}\left(\frac{\partial q_1}{\partial r_1}\right)_u dx$$
$$-\int_{\hat{x}}^{\bar{x}} x\frac{1}{q_2^3\bar{q}_2}\left(\frac{\partial q_2}{\partial r_2}\right)_u\left(\frac{1}{v_c}-\frac{1}{\bar{v}_c}\right)dx - \int_{\hat{x}}^{\bar{x}} x\frac{1}{q_2^2\bar{q}_2 v_c}\frac{\partial MRS_{qc}}{\partial c}\left(\frac{\partial q_2}{\partial r_2}\right)_u dx > 0,$$

because $(\partial q_i/\partial r_i)_u < 0$, $\partial MRS_{qc}/\partial c > 0$, and $\partial v_c/\partial x > 0$. Thus, since the numerator is positive and the denominator is negative in equation (A.11), we have $\partial\bar{x}/\partial f_2 < 0$.

Obtaining $\partial\bar{x}/\partial t_2$. Substitute $\lambda = t_2$ into equation (A.8), which becomes

$$\text{(A.12)} \qquad \frac{\partial\bar{x}}{\partial t_2} = \frac{-\int_0^{\hat{x}}\frac{x}{q_1^2}\frac{\partial q_1}{\partial u}\frac{\partial\bar{r}_2}{\partial t_2}dx + \int_{\hat{x}}^{\bar{x}}\frac{x}{q_2^2}\left(\frac{\partial q_2}{\partial t_2}\frac{\partial\bar{r}_2}{\partial u} - \frac{\partial q_2}{\partial u}\frac{\partial\bar{r}_2}{\partial t_2}\right)dx}{\frac{\bar{x}}{\bar{q}_2}\frac{\partial\bar{r}_2}{\partial u} + \int_0^{\hat{x}}\frac{x}{q_1^2}\frac{\partial q_1}{\partial u}\frac{\partial\bar{r}_2}{\partial x}dx + \int_{\hat{x}}^{\bar{x}}\frac{x}{q_2^2}\frac{\partial q_2}{\partial u}\frac{\partial\bar{r}_2}{\partial x}dx},$$

since $\partial q_1/\partial t_2 = \partial r_A/\partial t_2 = \partial L/\partial t_2 = 0$. The sign of the numerator is ambiguous as it stands. Substituting from equations (A.2), (A.3), and (A.4) into the numerator of equation (A.12), after some manipulation, produces

$$-\int_0^{\hat{x}} x\frac{(1-\alpha_1)x}{q_1^3\bar{q}_2}\left(\frac{\partial q_1}{\partial r_1}\right)_u dx - \int_0^{\hat{x}} x\frac{(1-\alpha_1)}{q_1^2\bar{q}_2 v_c}\frac{\partial MRS_{qc}}{\partial c}\left(\frac{\partial q_1}{\partial r_1}\right)_u dx$$
$$-\int_{\hat{x}}^{\bar{x}} x\frac{(1-\alpha_2)x}{q_2^3\bar{q}_2}\left(\frac{\partial q_2}{\partial r_2}\right)_u\left(\frac{1}{v_c}-\frac{1}{\bar{v}_c}\right)dx - \int_{\hat{x}}^{\bar{x}} x\frac{(1-\alpha_2)x}{q_2^2\bar{q}_2 v_c}\frac{\partial MRS_{qc}}{\partial c}\left(\frac{\partial q_2}{\partial r_2}\right)_u dx > 0,$$

because $(\partial q_i/\partial r_i)_u < 0$, $\partial MRS_{qc}/\partial c > 0$, and $\partial v_c/\partial x > 0$. Thus, since the numerator is positive and the denominator is negative in equation (A.12), we have $\partial\bar{x}/\partial t_2 < 0$

Obtaining $\partial\bar{x}/\partial\alpha_2$. Substitute $\lambda = \alpha_2$ into equation (A.8), which becomes

$$\text{(A.13)} \qquad \frac{\partial\bar{x}}{\partial\alpha_2} = \frac{-\int_0^{\hat{x}}\frac{x}{q_1^2}\frac{\partial q_1}{\partial u}\frac{\partial\bar{r}_2}{\partial\alpha_2}dx + \int_{\hat{x}}^{\bar{x}}\frac{x}{q_2^2}\left(\frac{\partial q_2}{\partial\alpha_2}\frac{\partial\bar{r}_2}{\partial u} - \frac{\partial q_2}{\partial u}\frac{\partial\bar{r}_2}{\partial\alpha_2}\right)dx}{\frac{\bar{x}}{\bar{q}_2}\frac{\partial\bar{r}_2}{\partial u} + \int_0^{\hat{x}}\frac{x}{q_1^2}\frac{\partial q_1}{\partial u}\frac{\partial\bar{r}_2}{\partial x}dx + \int_{\hat{x}}^{\bar{x}}\frac{x}{q_2^2}\frac{\partial q_2}{\partial u}\frac{\partial\bar{r}_2}{\partial x}dx},$$

since $\partial q_1/\partial\alpha_2 = \partial r_A/\partial\alpha_2 = \partial L/\partial\alpha_2 = 0$. The sign of the numerator is ambiguous as it stands. Substituting from equations (A.2), (A.3), and (A.4) into the numerator of equation (A.13), after some manipulation, produces

$$\int_0^{\hat{x}} x\frac{xt_2}{q_1^3\bar{q}_2 v_c}\left(\frac{\partial q_1}{\partial r_1}\right)_u dx + \int_0^{\hat{x}} x\frac{xt_2}{q_1^2\bar{q}_2 v_c}\frac{\partial MRS_{qc}}{\partial c}\left(\frac{\partial q_1}{\partial r_1}\right)_u dx$$
$$+\int_{\hat{x}}^{\bar{x}} x\frac{xt_2}{q_2^3\bar{q}_2}\left(\frac{\partial q_2}{\partial r_2}\right)_u\left(\frac{1}{v_c}-\frac{1}{\bar{v}_c}\right)dx + \int_{\hat{x}}^{\bar{x}} x\frac{xt_2}{q_2^2\bar{q}_2 v_c}\frac{\partial MRS_{qc}}{\partial c}\left(\frac{\partial q_2}{\partial r_2}\right)_u dx < 0,$$

because $(\partial q_i/\partial r_i)_u < 0$, $\partial MRS_{qc}/\partial c > 0$, and $\partial v_c/\partial x > 0$. Thus, since the numerator and denominator are negative in equation (A.13), we have $\partial\bar{x}/\partial\alpha_2 > 0$.