Games on Cellular Spaces

Pedro Ribeiro de Andrade, Antônio Miguel Vieira Monteiro, Gilberto Câmara

INPE – National Institute for Space Research
Av. dos Astronautas 1758, CEP 12227-001, São José dos Campos, Brazil
{pedro, miguel, gilberto}@dpi.inpe.br

The role of spatial structures in the study of games has drawn a lot of attention in the last years. Games with spatially explicit factors have proven to be useful for modelling biological and economic environments [1]. The purpose of such games is to assess the effects that spatial structures have on adaptation strategies of agents, mainly in the study of the evolution of altruistic behaviour. Adding a spatial component to the models often displays different features from models with well-mixed populations because it changes the way with which agents interact with each other. For example, the evolution of interspecific mutualism cannot be explained by an unstructured population through the iterated continuous Prisoner's Dilemma [2].

In this work, we study the effects of a spatial structure in the development of a model where players have three main characteristics: *limited resources*, *no rationality*, and *mobility*. Due to the difficulty to deal with these points simultaneously in a purely mathematical model, we decided to build an agent-based model to simulate competitions for space.

The proposed model uses the non-cooperative game C_X, shown in Table 1. C_X applies X to the payoffs where one player escalates and the other does not. Given that this game has only two pure strategies, we say that s_k, $0 \leq k \leq 1$, is the mixed strategy of escalating with probability k. This game is symmetric, and the equilibrium point of a game C_X where both players apply the same mixed strategy is $s_{X/10}$. The objective of this work is to study the relations between the results of the proposed model and this equilibrium strategy.

Table 1. Payoffs of game C_X, in pairs (A, B).

	B escalates	*B* does not escalate
A escalates	(−10, −10)	(+X, −X)
A does not escalate	(−X, +X)	(0, 0)

The model takes place in a cellular space. A cellular space is a network of cells populated with agents and connected by neighbourhood relations. Each agent belongs to a cell, which has enough space for it to live. Initially, a cell contains a set of agents, which have to compete for it through the game above. Whenever an agent is playing a game, we call it *player*.

The basic assumption of this model is that whenever an agent arrives at a cell it is satisfied with it, and it will not move until it becomes dissatisfied. Two agents within the same cell may play a game competing for it, and the result of the game affects the satisfaction of both. This is the only memory an agent has, and it is called *local satisfaction*. It starts with a positive value when an agent arrives at a cell, and when this value reaches zero or less, the agent randomly picks a neighbour cell and moves to it, looking for a better cell to compete for. Each agent also has a *global satisfaction* representing its initial resources, starting with a positive value significantly greater than the local satisfaction, but also affected by the payoffs of the games. All agents have the same global satisfaction at the beginning of the model. An agent that got dissatisfied many times and its resources end leaves the model. As we need local and global satisfaction decreasing along the simulation, the expected payoff of the game used in the model has to be almost always negative. Agents are identical if we consider satisfaction, but they differ in their mixed strategies, which cannot be identified by any other agent.

This model has a finite number of turns, each one with two steps. The first step establishes the games, randomly choosing pairs of agents in each cell, and then carries out games with each pair. Cells with an odd number of agents have one random idle agent. No agent plays more than once in each turn. Each selected agent chooses a pure strategy randomly based on its mixed strategy. The second step defines the dynamical part of the model. Once each agent already knows its own payoff, it updates its local and global satisfactions with the earned payoff, and then it checks if any satisfaction has reached zero or less to relocate or to leave the model. The model executes until it reaches a stable state, which can be when there is at most one agent in each cell, or when the overall satisfaction of the model stops to decrease.

An example of executing this model is shown in Figure 1. The simulations apply C_1 in a 20x20 squared cellular space, where the possible movements are at most to four neighbours (up, down, left, and right). The model started with 3 agents following each of eleven mixed strategies {$s_{0.0}$, $s_{0.1}$, ..., $s_{1.0}$.} in each cell, summing up 13200 agents, each one having 200 as global satisfaction and 20 as local satisfaction. The graphic shows the number of cells owned by agents of different strategies along 50 simulations. A cell is owned by the agent within it that has higher global satisfaction, meaning that it has more chance to survive than the others. The strategies from $s_{0.8}$ to $s_{1.0}$ have a development similar to $s_{0.7}$ and were omitted. We can see that each single strategy has similar results in all simulations. At the end, the more successful strategy was $s_{0.2}$, the second one was $s_{0.3}$, and the third was the analytical equilibrium point ($s_{0.1}$). Also, there is not much difference between never escalating and acting without any strategy ($s_{0.5}$), but it is better acting cooperatively than to adopt a tendency to escalate more often than not to escalate.

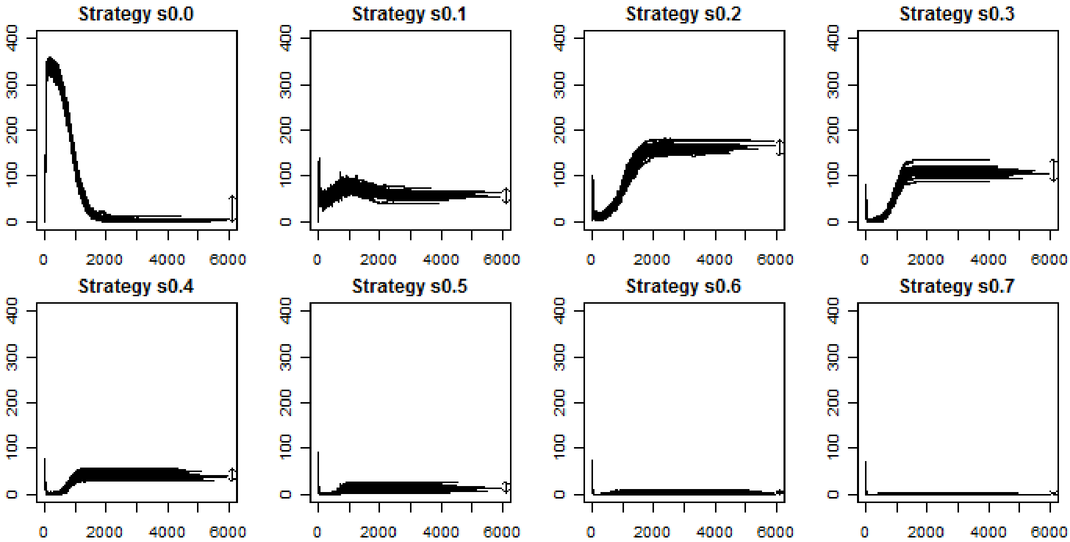
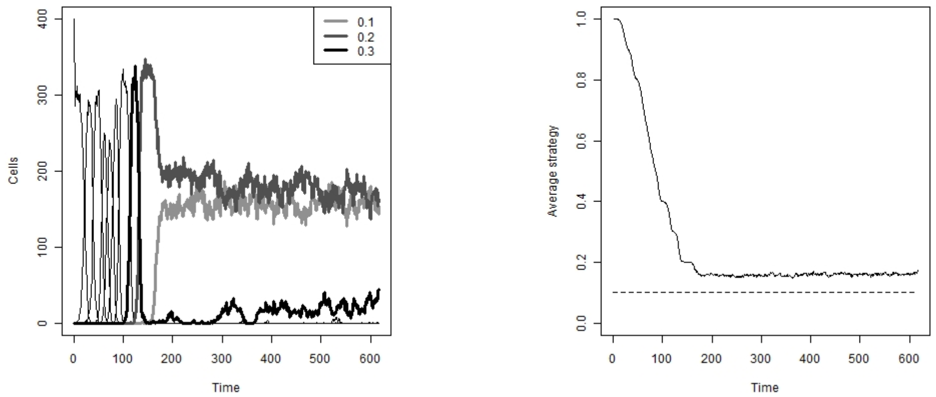


Figure 1. Ownership of strategies along 50 simulations.

The results of Figure 1 can be explained by the fact that the more an agent escalates, it can realize an unfavourable arrangement earlier, leaving the cell faster than other strategies. The counterpart of escalating more is the higher destruction of agents within the same strategy. Nash equilibrium is the base for a stable relation, and it can be used as a starting point for the best strategy. In this model, the best strategy uses the equilibrium, but it adds some risk to get more mobility. The winning strategy mixes both characteristics; its agents are not exploited by threatening agents and can conquer cells from other agents.

From this model with a single population, we developed an evolutive model. It is based on the idea that agents that own cells at the end of the original model deserve to pass their strategies to the next generation. Therefore, the new model considers the previous one as one evolutive step, a *survival of the fittest*. Once a competition finishes, each agent that has conquered a partition of space produces descendants within the cell it has conquered. The agent tries to transfer its own strategy to its offsprings, but their learning ability is limited, and then they may have a strategy slightly different from the father, according to a predefined mutation probability. Whenever this mutation is activated, the offspring has the strategy of its father with some random change; otherwise it takes exactly the same strategy.

Figure 2(a) depicts the evolution of the strategies in a model that starts with a population of $s_{1.0}$ and has mutation change of ±0.1. Along the simulation, new strategies emerge through mutation and dominate the cellular space until a new strategy emerge and so on, until the three more successful strategies ($s_{0.1}$, $s_{0.2}$, and $s_{0.3}$, as in the results above) appear. However, none of them can surpass the other two strategies. Although the number of agents following each specific strategy never stabilizes, the model converges to a stable state. In Figure 2(b) we can see that the average strategy of the population stabilizes above the theoretical equilibrium point ($s_{0.1}$, drawn as a dashed line).



(a) Cells owned by different strategies (b) Convergence of the average strategy

Figure 2. Evolutive simulations with C_1 and initial population of $s_{1.0}$.

Finally, we analysed a set of different games, from $C_{0.2}$ to $C_{9.8}$, in order to study a range of equilibrium points using different configurations of C_X. The results are shown in Figure 3. The final distribution of strategies in the range of equilibrium points is slimilar to a logistic equation, where the cooperative pure strategy (not to escalate) attracts and the more defective one repulses the population. The standard deviation also increases as the equilibrium point moves towards the pure strategy of always escalating. The dashed line shows the points where the average strategy would be equal to the analytical equilibrium point.

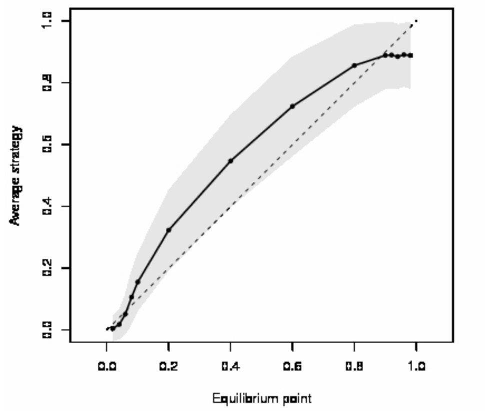


Figure 3. Evolution of strategies under different equilibrium points.

The results of our experiments show that, even with complete irrational agents that follow the same mixed strategies during their lifetime, the average strategy of the population always converges to a stable state, close to the analytic equilibrium point. This point corroborates the hypothesis that populations evolve to a stable state even if we add more realistic components to a model that works with non-cooperative games. The results also agree with the statements of Nash, who said that "we can only expect some sort of approximate equilibrium, since […] the stability of the average frequencies will be imperfect" [3]. A better description of the models presented in this paper and other results can be found in [4, 5].

References

1. Nowak, M.A. and K. Sigmund, *Games on Grids*, in *The Geometry of Ecological Interactions: Simplifying Spatial Complexity*, U. Dieckmann, R. Law, and J.A.J. Metz, Editors. 2000, Cambridge University Press. p. 135-150.
2. Scheuring, I., *The iterated continuous prisoner's dilemma game cannot explain the evolution of interspecific mutualism in unstructured populations.* Journal of Theoretical Biology, 2005. **232**: p. 99-104.
3. Nash, J., *Non-cooperative games*, in *Mathematics Department*. 1950, Princeton University.
4. Andrade, P.R., et al., *Games on Cellular Spaces: How Mobility Affects Equilibrium*, in *Journal of Artificial Societies and Social Simulation*. 2009. p. 5.
5. Andrade, P.R., A.M.V. Monteiro, and G. Camara, *Games on Cellular Spaces: An Evolutionary Approach*, in *Progress in Artificial Intelligence, 14th Portuguese Conference on Artificial Intelligence, EPIA 2009*, L. Seabra Lopes, et al., Editors. 2009, Springer Verlag: Aveiro, Portugal. p. 535-546.